An Inverse Finite Element Method for Application to Structural Health Monitoring

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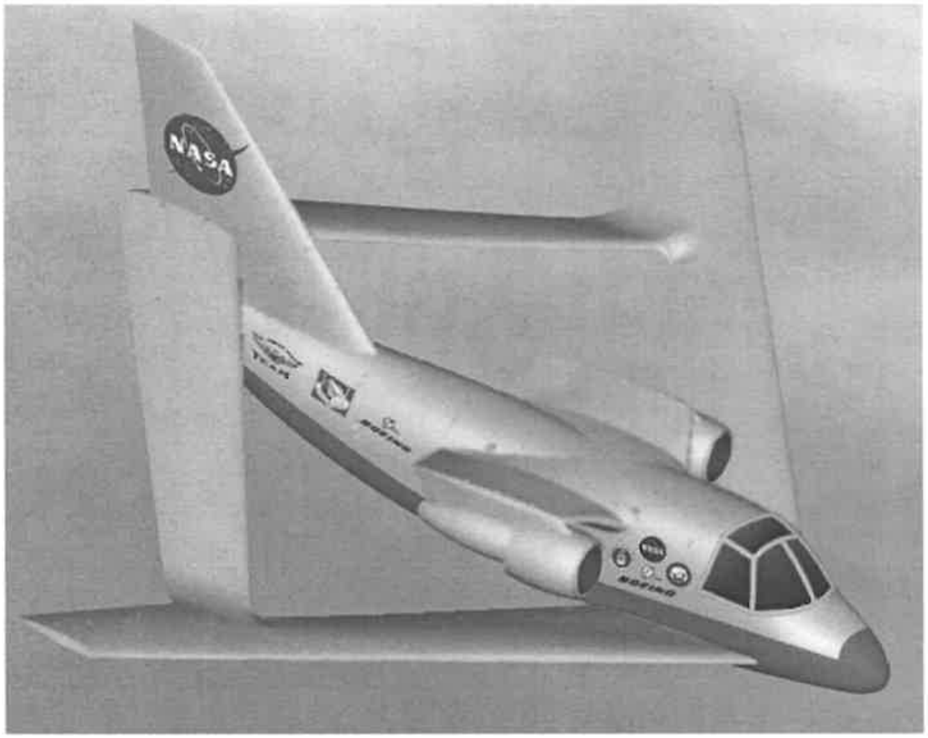
Motivation

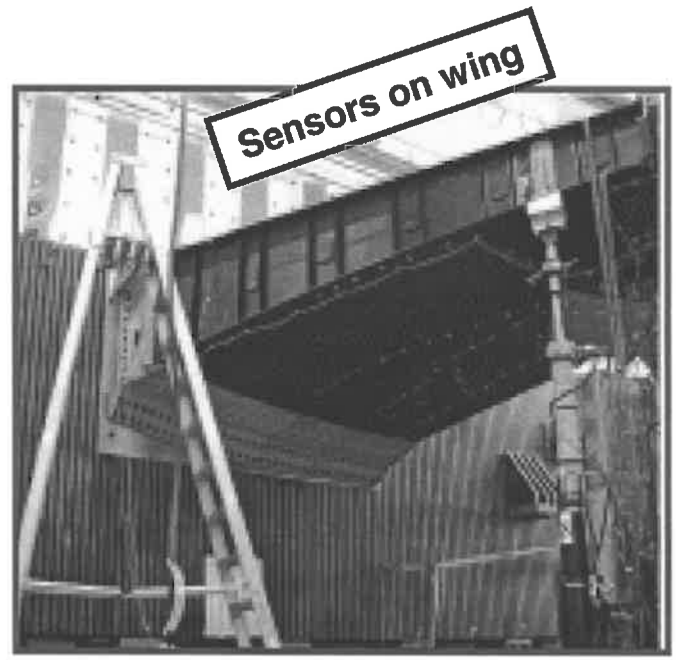


- Next generation of aerospace/aircraft vehicles
  - Multifunctional structures
  - Fiber Optics (FO) sensor system
  - Structural health monitoring in real time
- Morphing wing technology
  - deformations by embedded actuators
- "Joined Wing" aircraft
  - Radar on wing surfaces
  - Wing deflections
  - Radar adjustment

Inverse Problem

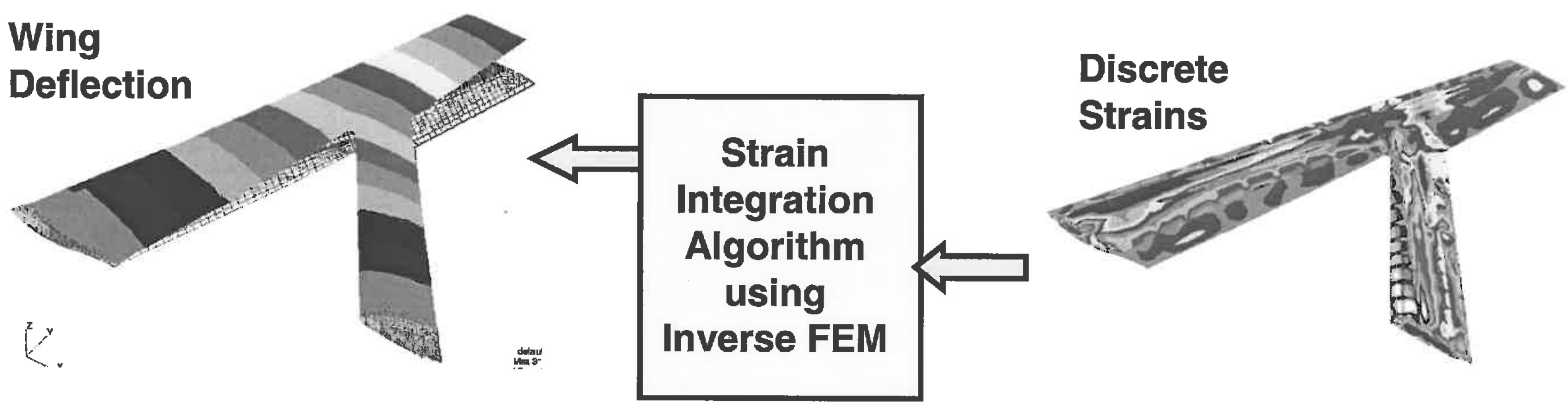
- *Strain sensors on surfaces* of structural components
- Arbitrary *location and orientation*
- *Objective:*

  Determine deformed shape of structure in real time



Approach

Develop *Inverse FEM* for *reconstruction of deformation* using measured strain data, i.e., integrate ε-**u** relations to obtain **u**



Background

- Tikhonov-Arsenin (1977)
  - Ill-posed, inverse problems, regularization method
- Tessler-Dong ('81), Tessler-Hughes ('85)
  - Anisoparametric C^0 elements
- Tessler et al ('93)
  - Discrete least-squares finite elements for smoothing of data

Variational Formulation of Inverse FEM Element level

- Find an extremum of the smoothing functional for a fixed value of the regularization parameter λ :

$$\Phi^{\lambda}(\mathbf{u}^h) = \left\| \boldsymbol{\varepsilon}(\mathbf{u}^h) - \boldsymbol{\varepsilon}^{\delta} \right\|^2 + \left\| \boldsymbol{\kappa}(\mathbf{u}^h) - \boldsymbol{\kappa}^{\delta} \right\|^2 + \lambda \left\| \boldsymbol{\gamma}(\mathbf{u}^h) - \boldsymbol{\gamma}^{\delta} \right\|^2$$

$$\left\| \boldsymbol{\varepsilon}(\mathbf{u}^h) - \boldsymbol{\varepsilon}^{\delta} \right\|^2 \equiv \frac{1}{n} \sum_{i=1}^{n} \left[\boldsymbol{\varepsilon}(\mathbf{u}^h)_{\mathbf{x}_i} - \boldsymbol{\varepsilon}_i^{\delta} \right]^2$$

Euclidean squared norm in terms of membrane strains

$$\left\| \boldsymbol{\kappa}(\mathbf{u}^h) - \boldsymbol{\kappa}^{\delta} \right\|^2 \equiv \frac{\Omega^e}{n} \sum_{i=1}^{n} \left[\boldsymbol{\kappa}(\mathbf{u}^h)_{\mathbf{x}_i} - \boldsymbol{\kappa}_i^{\delta} \right]^2$$

Norm in terms of bending curvatures

$$\left\| \boldsymbol{\gamma}(\mathbf{u}^h) - \boldsymbol{\gamma}^{\delta} \right\|^2 \equiv \frac{1}{n} \sum_{i=1}^{n} \left[\boldsymbol{\gamma}(\mathbf{u}^h)_{\mathbf{x}_i} - \boldsymbol{\gamma}_i^{\delta} \right]^2$$

Norm in terms of transverse shear strains

$\boldsymbol{\varepsilon}_i^{\delta}, \quad \boldsymbol{\kappa}_i^{\delta}, \quad \boldsymbol{\gamma}_i^{\delta}$ Arrays of discrete measured strains at $\mathbf{x}_i$

Strain-displacement relations

$$\boldsymbol{\varepsilon} \equiv \begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & 0 & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \end{Bmatrix}$$

$$\mathbf{u}^h \equiv \mathbf{N}\,\mathbf{d}$$

C^0 interpolated displacements

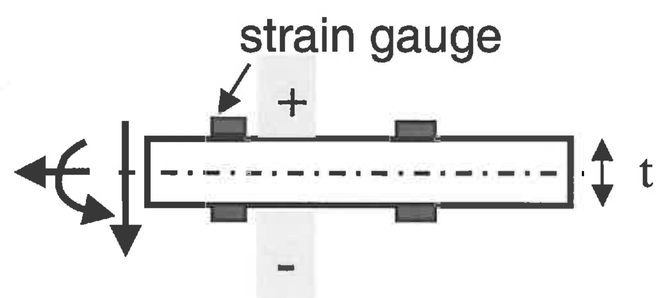
$$\boldsymbol{\kappa} \equiv \begin{Bmatrix} \kappa_{xo} \\ \kappa_{yo} \\ \kappa_{xyo} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \end{Bmatrix}$$

$$\boldsymbol{\gamma} \equiv \begin{Bmatrix} \gamma_{xzo} \\ \gamma_{yzo} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial x} & 0 & 1 \\ 0 & 0 & \frac{\partial}{\partial y} & 1 & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \end{Bmatrix}$$

Measured strains

$$\varepsilon_{xo}^{\delta} = \frac{1}{2}(\varepsilon_{xx}^{+} + \varepsilon_{xx}^{-}),$$

$$\kappa_{xo}^{\delta} = \frac{1}{t}(\varepsilon_{xx}^{+} - \varepsilon_{xx}^{-})$$



Linear Equations

Minimize element smoothing functional

$$\Phi^{\lambda}(\mathbf{u}^h) = \left\| \boldsymbol{\varepsilon}(\mathbf{u}^h) - \boldsymbol{\varepsilon}^{\delta} \right\|^2 + \left\| \boldsymbol{\kappa}(\mathbf{u}^h) - \boldsymbol{\kappa}^{\delta} \right\|^2 + \lambda \left\| \boldsymbol{\gamma}(\mathbf{u}^h) - \boldsymbol{\gamma}^{\delta} \right\|^2$$

And summing on all elements results in

$$\mathbf{K}\,\mathbf{d} = \mathbf{F}$$

Ultra-fast solution

$\mathbf{K} \equiv \mathbf{K}(\mathbf{x}_i)$ Symmetric and positive definite

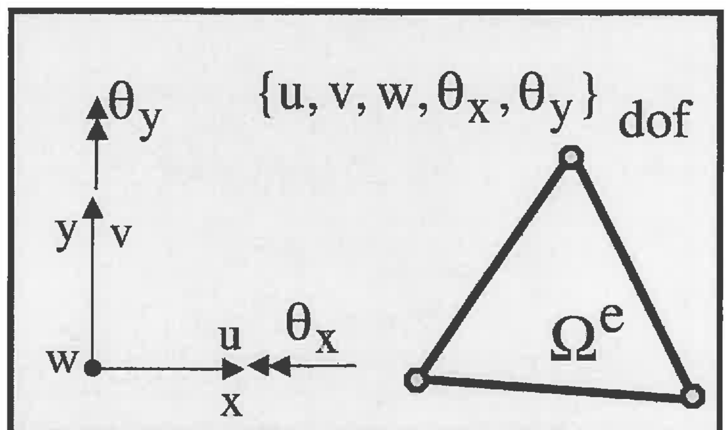
$\mathbf{d} \equiv \mathbf{d}(\mathbf{u})$ Vector of displacement dof's

$\mathbf{F} \equiv \mathbf{F}(\boldsymbol{\varepsilon}^{\delta})$ R.h.s. “load” vector

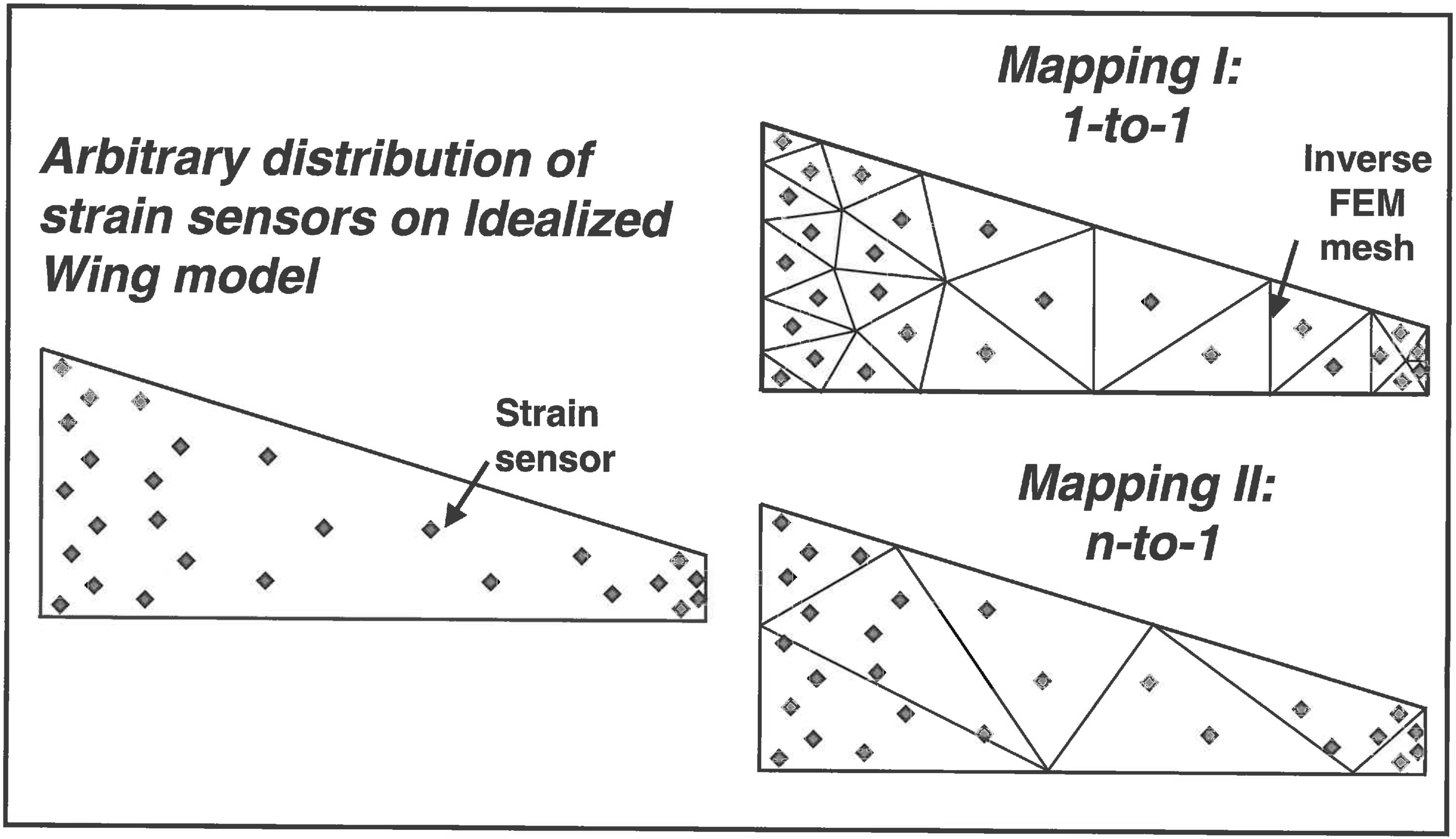
MIN3 and its Inverse Element, $MIN3^{-1}$

- **Direct FEM**: MIN3, 3-node plate ('85)
- C^0 kinematic interpolations
  - u, v, θ_x, θ_y -- linear
  - w -- anisoparametric quadratic
  - Membrane and bending strains constant
- Principle of Min. Potential Energy:
  - Minimize: Π
  - $\phi^2 K_{shear}$ (MIN3*)

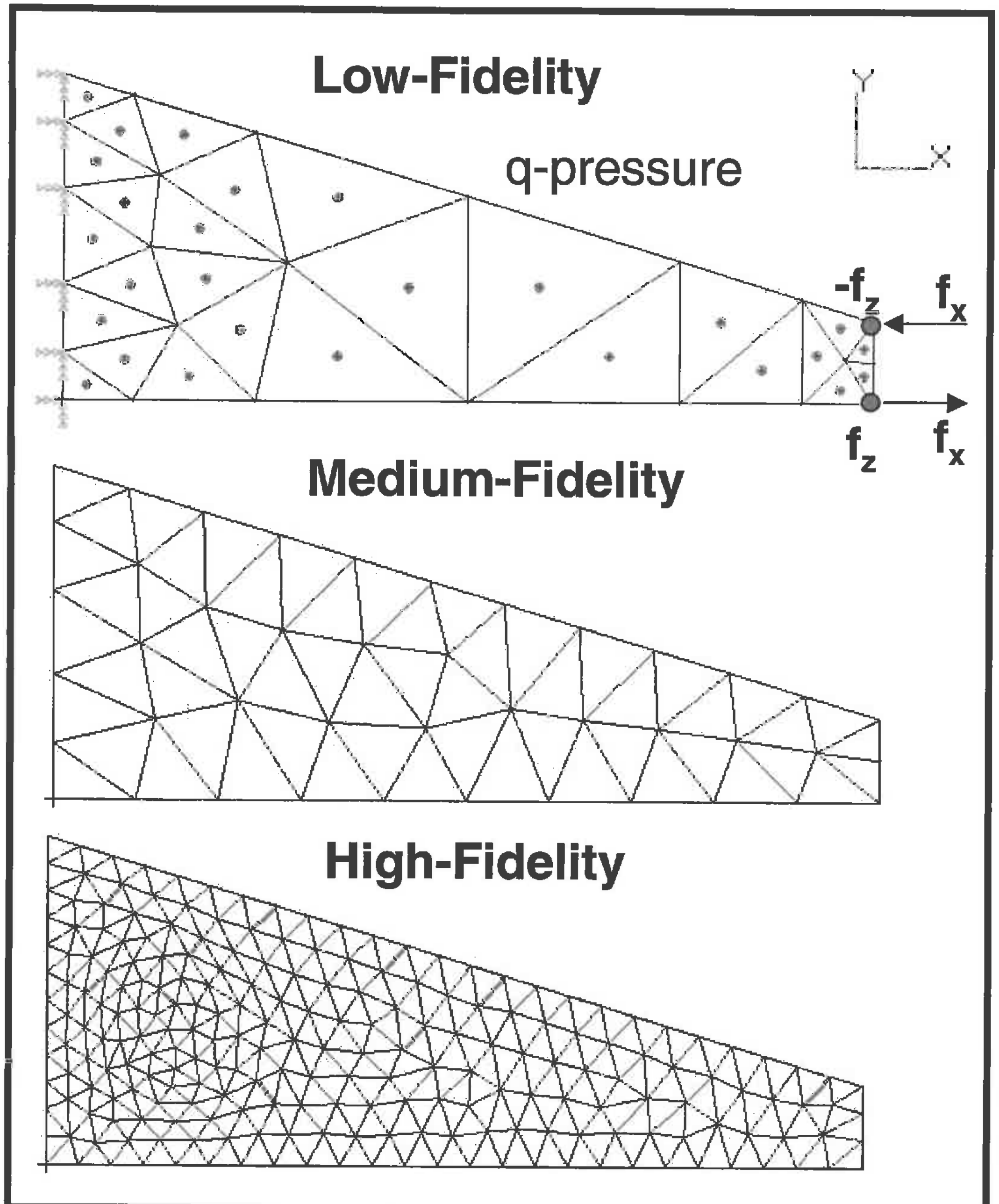
- **Inverse FEM:** $MIN3^{-1}$, 3-node element
- C^0 kinematic interpolations as MIN3
- Least squares formulation:
  - Minimize: Φ
  - Loads and materials unspecified



Inverse FEM: Mapping of strain data

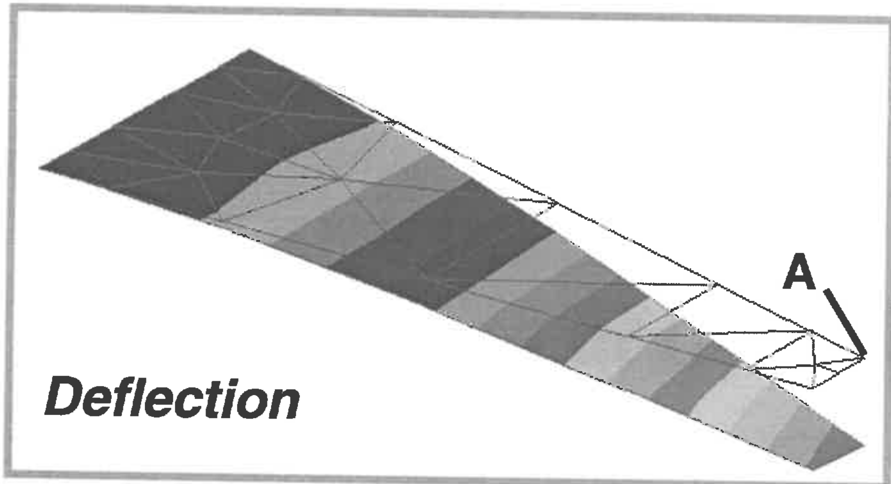


Numerical Experiment: Idealized Wing Model

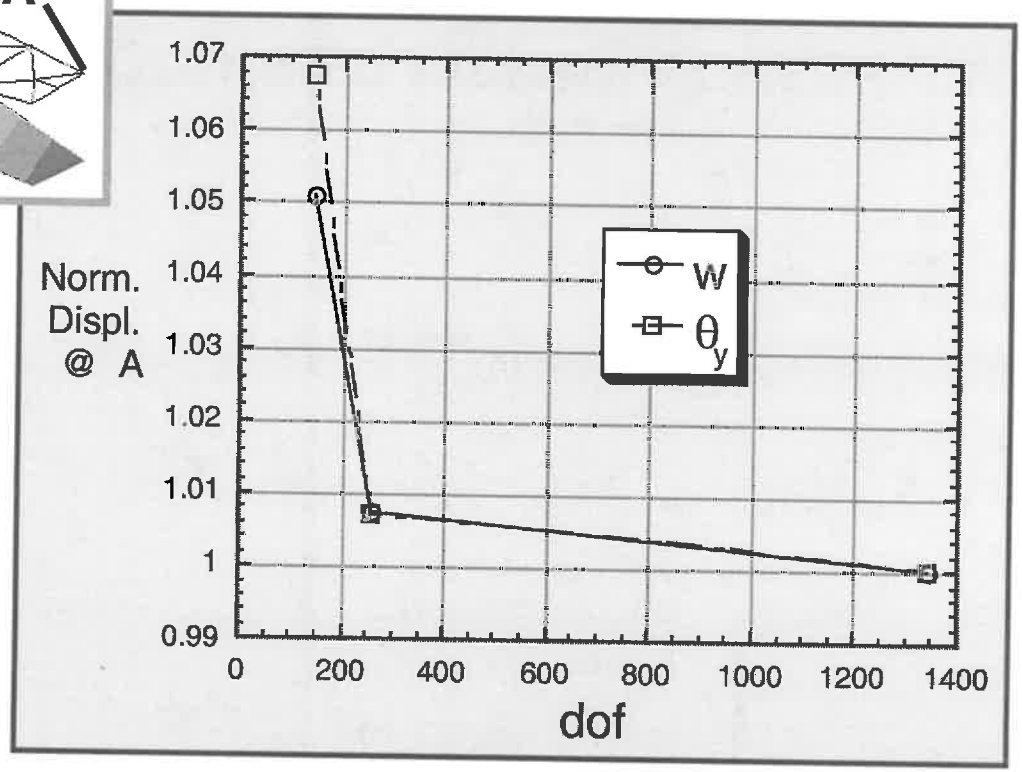


- Aluminum panel clamped at left end
- Loads
  - Uniform pressure
  - Twisting forces
  - In-plane forces
- Ultra-thin plate: span/thickness=$6*10^4$

FEM (MIN3):*
Convergence of Deflection and Bending Rotation

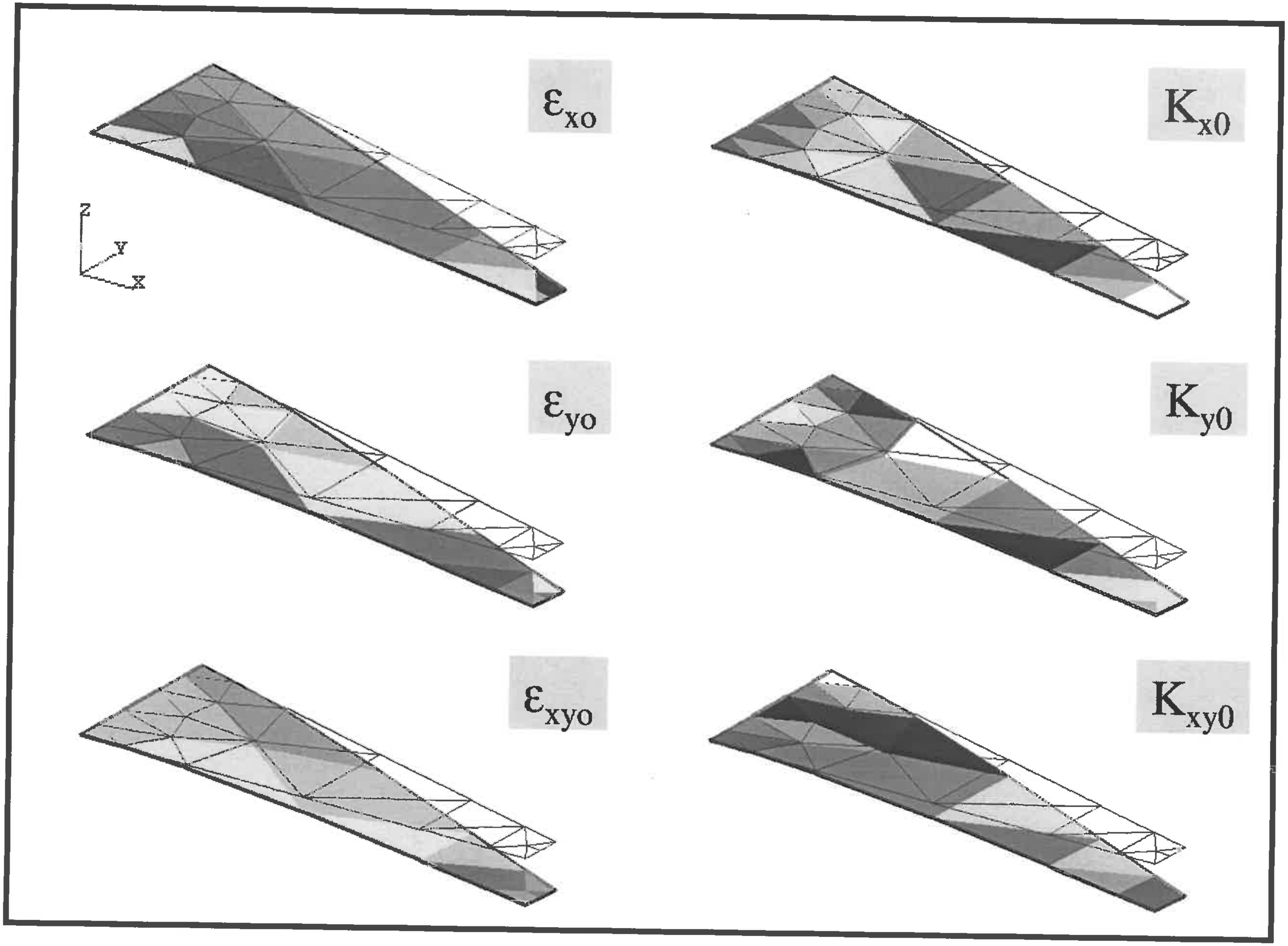


Results @ node A



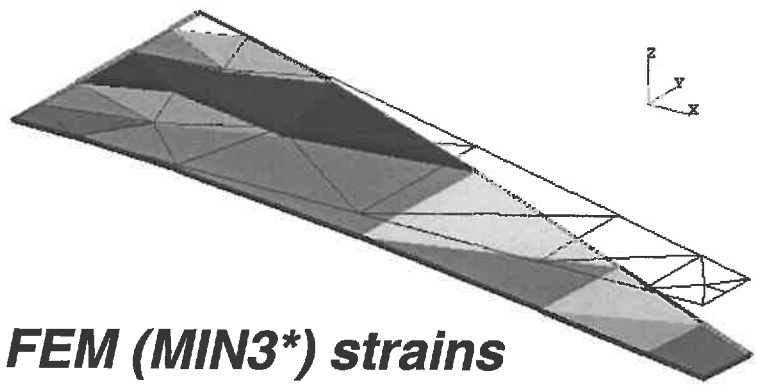
- MIN3* strains will be used to represent experimental data

Low-Fidelity FEM (MIN3*) Strain Distributions, i.e. Low Quality “Experimental Data”

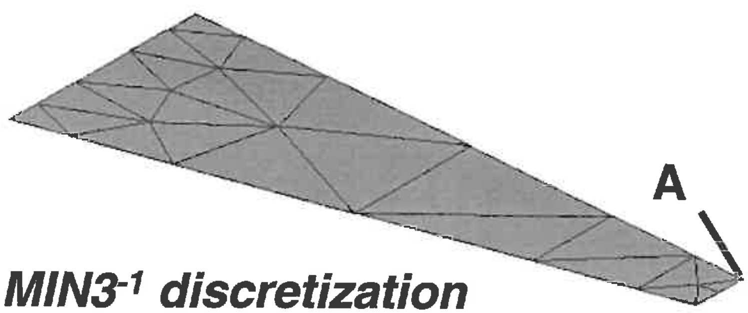


$MIN3^{-1}$ Displacement Reconstruction from MIN3 strain data*

Mapping I (1-1)

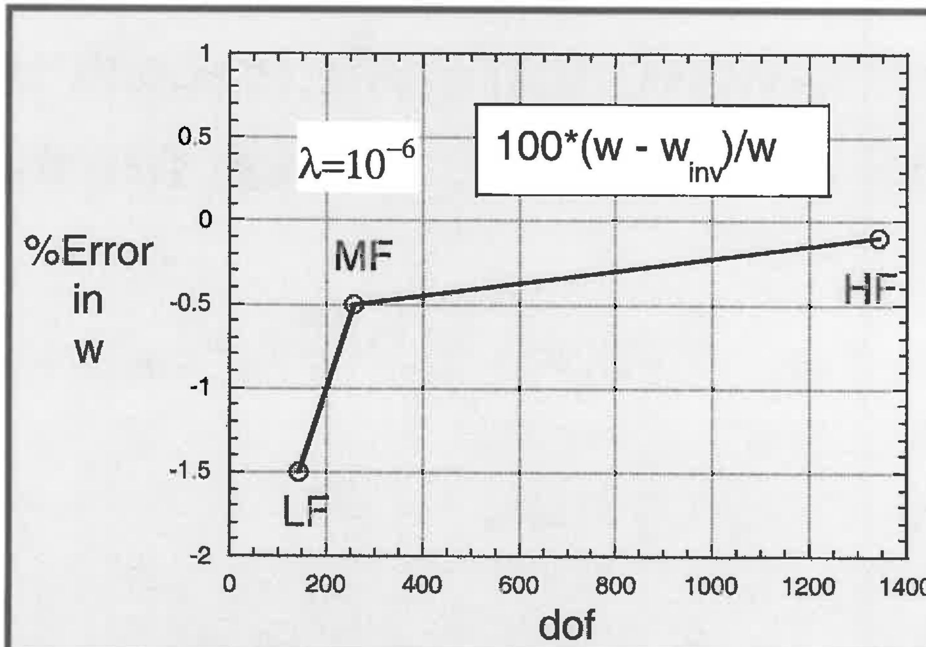


FEM (MIN3*) strains



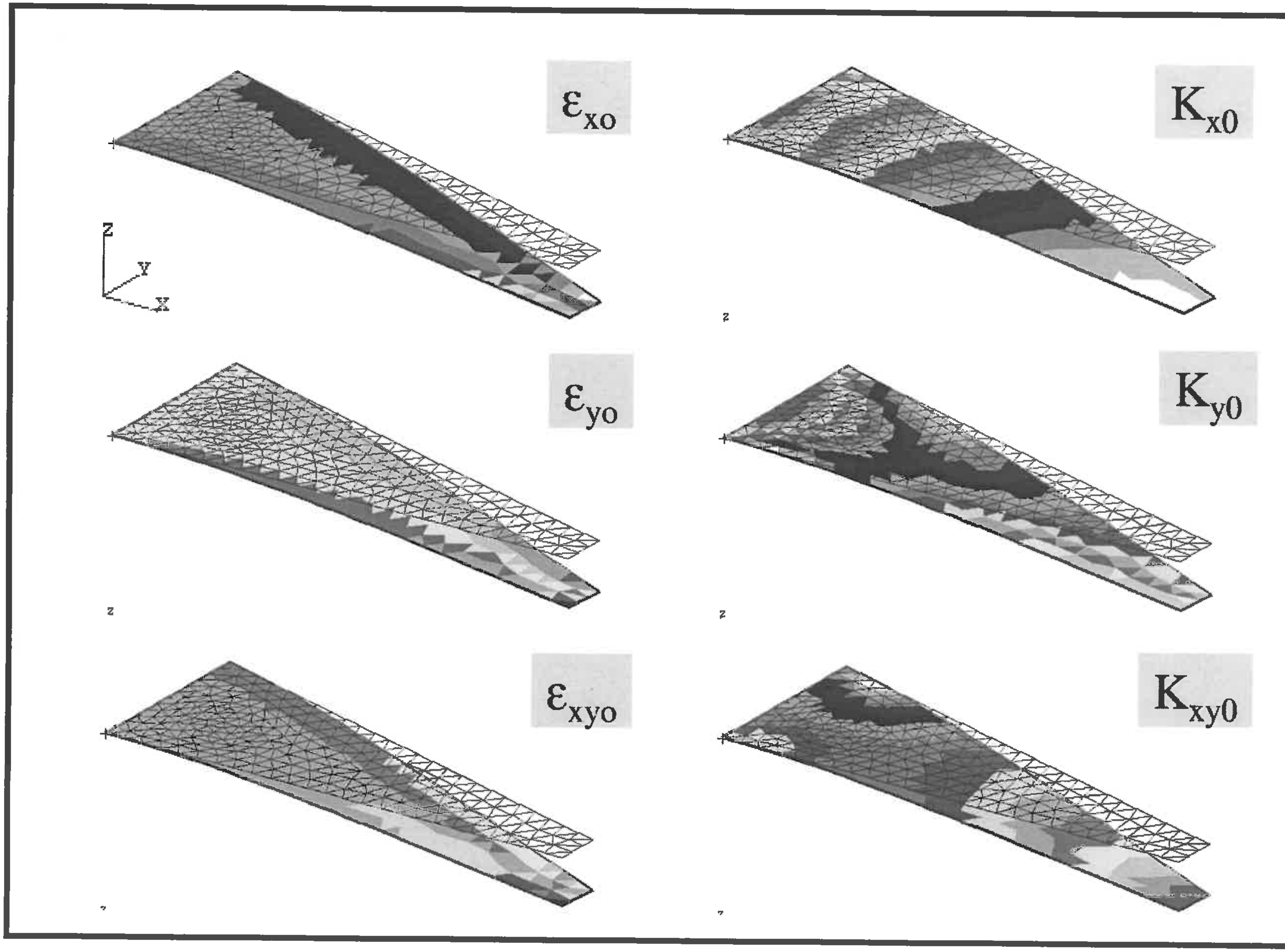
$MIN3^{-1}$ discretization

Results @ node A



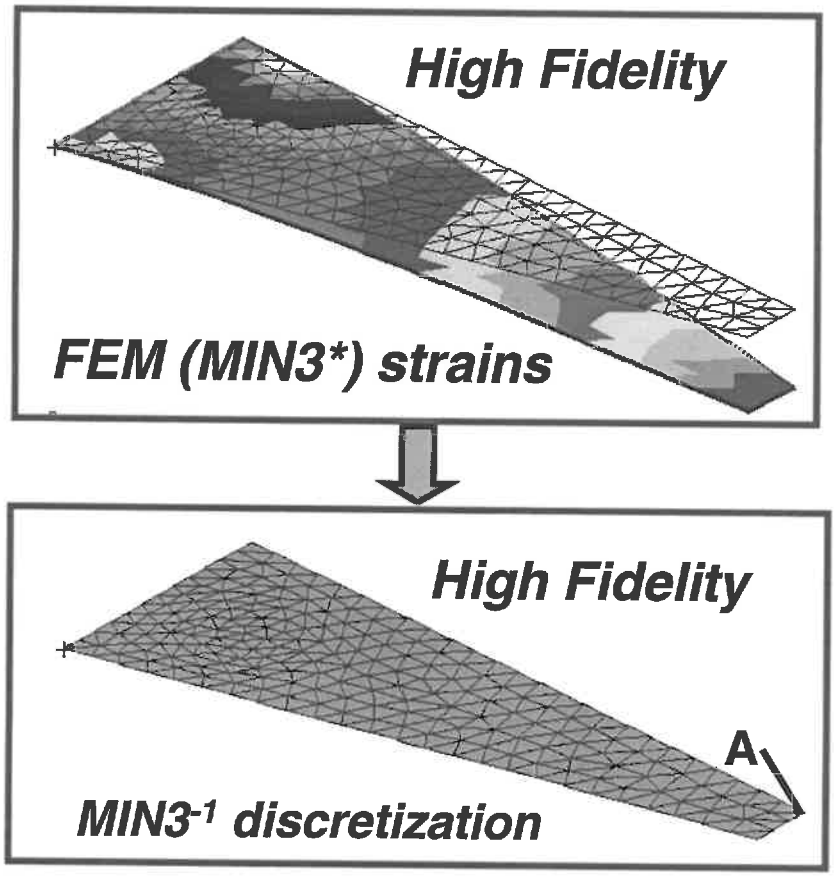
- All other displacement variables reconstructed precisely across panel

High-Fidelity FEM (MIN3) Strain Distributions. i.e. Accurate "Experimental Data"*

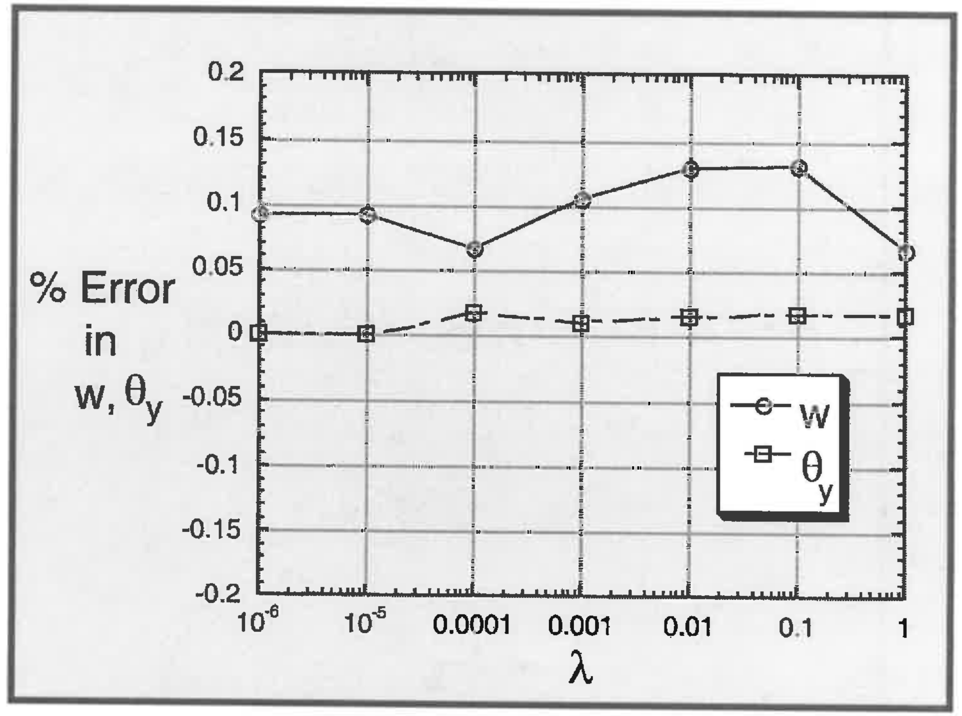


Study of Lambda Parameter

Mapping I (1-1)



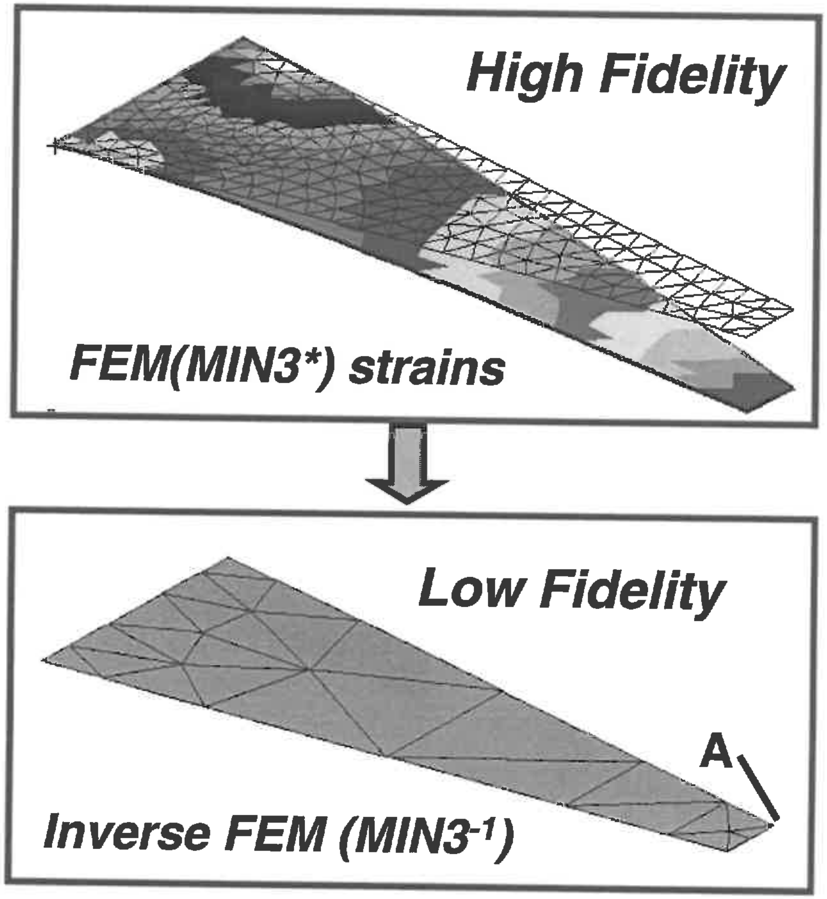
Results @ node A



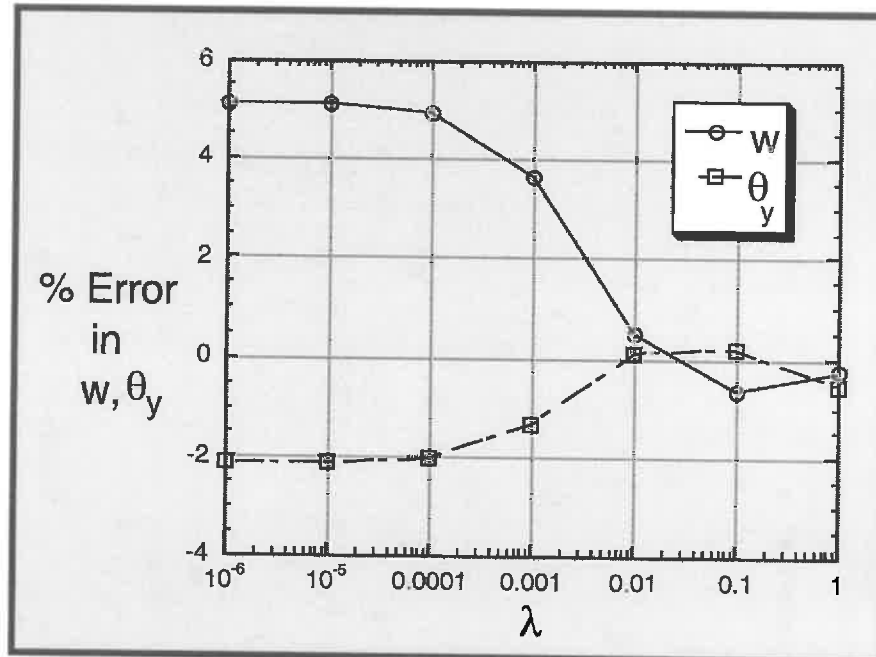
- In-plane displacements, u and v, reconstructed precisely across panel

Study of Lambda Parameter

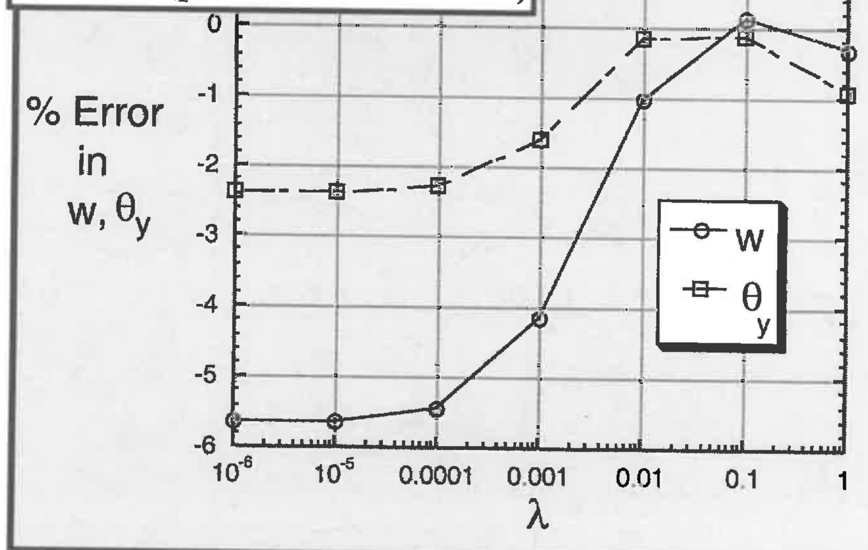
Mapping II (n-1)



Results @ node A



$\varepsilon^{\delta} = \varepsilon * (1 + 0.05 * \delta)$,
$\delta \in [-1, 1]$ (pseudorandom number)



Remarks on Numerical Results

- $MIN3^{-1}$ is exact inverse of MIN3
  - In 1-1 mapping of MIN3 strains, $MIN3^{-1}$ reconstructs all displacements precisely
- MIN3* more accurate in bending than MIN3
- Shear locking eliminated via anisoparametric interpolation
- Mapping I (1-1)
  - Effect of Lambda is insignificant
- Mapping II (n-1)
  - Serves as filter/smoother
  - "Optimal" Lambda
  - Strains with random error "smoothed" effectively

Concluding Remarks

- Standard FEM architecture
- Arbitrary topology application
- Static and dynamic problems
- Small and large displacements (incremental linear approach)
- Thin and thick beams, plates, shells, solids and built-up structures
- Sensor location optimization
- Ultra-fast computations