



# Advances in Structural Analysis Methods for Structural Health Management of NextGen Aerospace Vehicles

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NASA Langley Research Center

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# Outline

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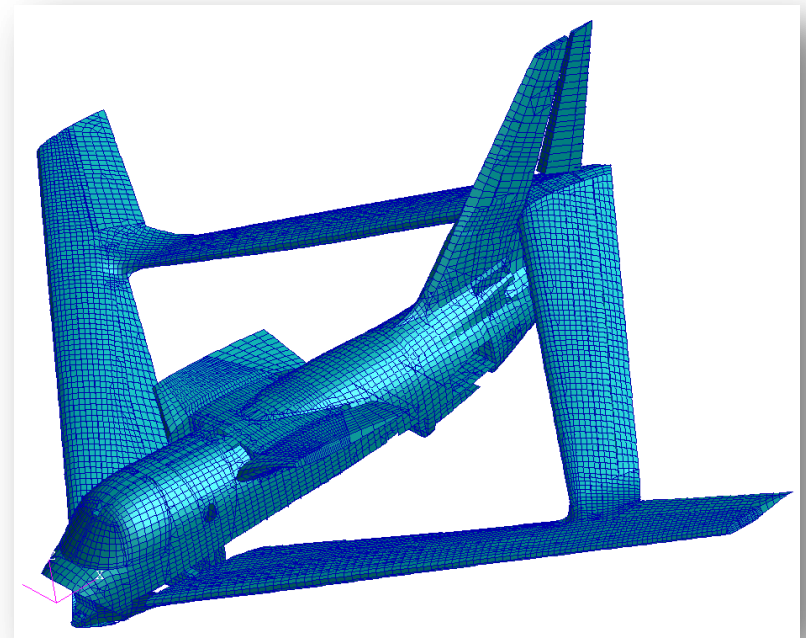
- Motivation
- Vehicle Health Management
- Shape-sensing
- Shape-sensing (NASA Dryden)
- Full-field reconstruction (NASA LaRC)
- Collaborations
- Summary

# Motivation: Sensing of wing deformations



FBG strain sensing – wing deformation (inverse reconstruction, ill-posed problem)

Conforming antenna on  
AEW&C (airborne radar system)



Strain-displacement relations

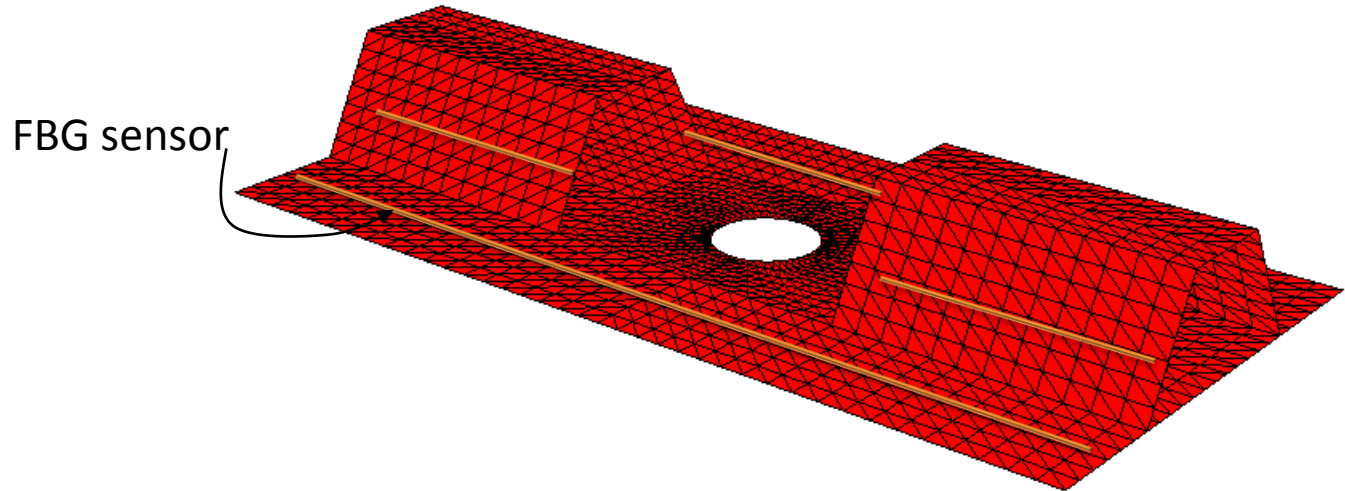
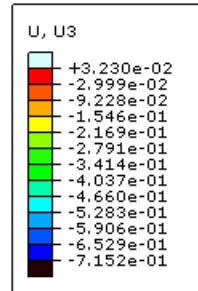
$$\boldsymbol{\varepsilon}^h = \mathbf{L} \mathbf{u}^h \text{ on } \Omega^h$$

# Shape sensing: from in-situ strains to deformed shape

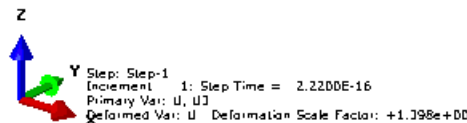


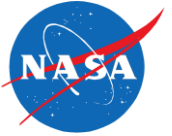
Scale Factor: +0.00

Hat-stiffened panel:  
full-field solution



ABAQUS job created on 09-Oct-03 at 17:25:04  
ODB: HSP\_HighFi\_LinearFEM\_LocalNadal.odb Abaqus/Standard 6.10-1 Tue Mar 15 12:29:04 Eastern Daylight Time 2011





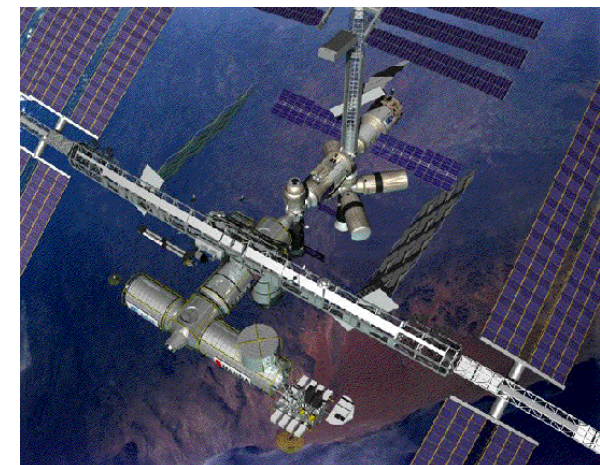
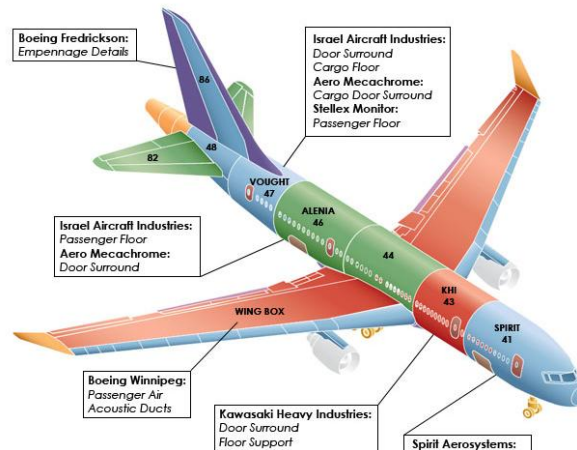
## Objectives

- Affordable, safe and reliable technologies for aeronautic and long-duration space structures
  - Provide real-time vehicle health information via sensors, software and design by monitoring critical structural, propulsion, and thermal protection systems
  - Provide valuable information to adaptive control systems to mitigate accidents due to failure and achieve safe landing
  - Provide detection and localization of impact events on key structural and flight control surfaces
  - Utilize decision-making mechanisms using intelligent reasoning based on safe-outcome probability
  - Maximize performance and service life of vehicle or space structure

# Continuous Monitoring and Assessment of Structural Response in Real Time



- Diagnosis and prognosis of structural integrity
  - Deformation
  - Temperature
  - Strains and stresses (internal loads)
  - Damage and failure

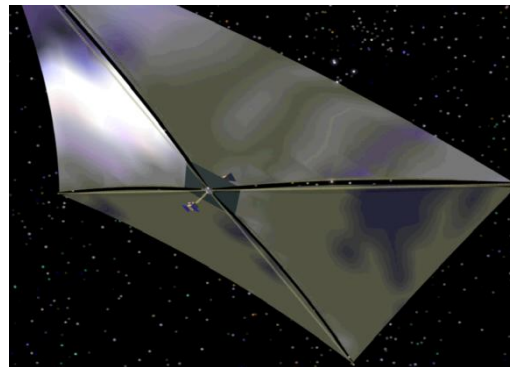
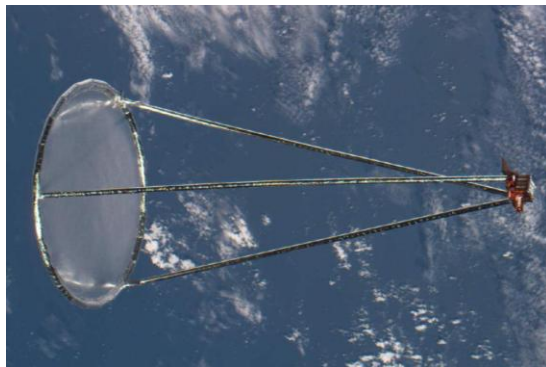


# Maximize Performance: Provide Active Structural Control via Shape Sensing



- Helios class of aircraft (solar panel)
  - Control of wing dihedral
- Unmanned Aerial Vehicles (UAV)
- Morphing capability aircraft
  - Shape changes of aircraft wing
- Embedded antenna performance
- Shape control of large space structures
  - Solar sails
  - Membrane antennas

## Wing control systems



Shape Control of Space Structures



# Implementation & enabling capabilities

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- Diverse arrays of distributed in-situ sensors
  - Process, communicate, and store massive amounts of SHM data
  - Perform on-board structural analysis based on SHM sensing data
    - Determine deformed shape of structure continuously
    - Perform diagnosis and prognosis of structural integrity
  - Provide information of structural integrity to cockpit displays and remote monitoring locations to enable safe and effective operational vehicle management and mission control
  - Provide valuable information to improve future designs



# NASA Dryden Shape-Sensing Analysis



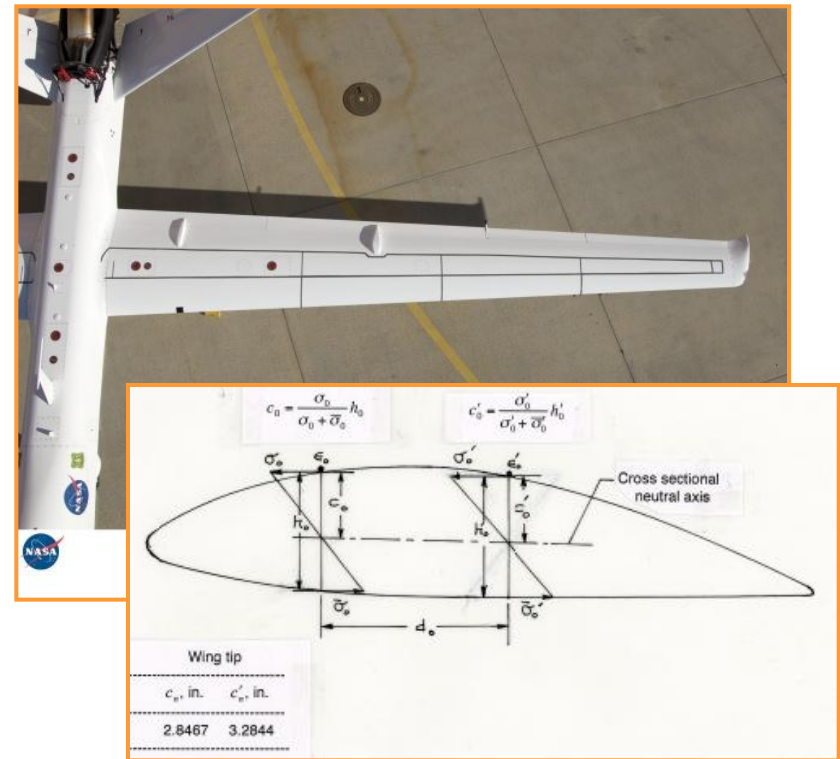
- 1-D integration of classical beam Eqs for cantilevered, non-uniform cross-section beams (no shear deformation)

$$w_{,xx} = \frac{\varepsilon_x^+}{-c(x)} \quad (u_x(x, z) = -z w_{,x})$$

$$z \in [c, -c]$$

- Piecewise linear approximation of strain and taper between regularly spaced “nodes” where strains are measured
- Neutral axis is computed from detailed FEM (SPAR code)
- Incorporates cross-sectional geometry of a wing in a beam-type approximation

View from above the left wing  
(Optical fiber is glued on top of wing)



*Method for Real-Time Structure Shape-Sensing, U.S. Patent No. 7,520,176, issued April 21, 2009.*

# NASA Dryden Shape-Sensing Analysis

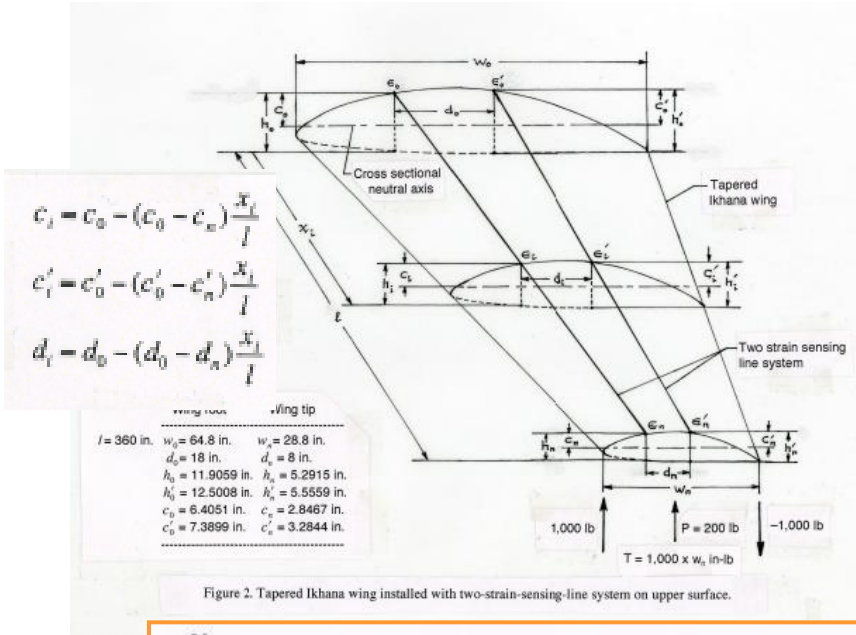


Figure 2. Tapered Ikhana wing installed with two-strain-sensing-line system on upper surface.

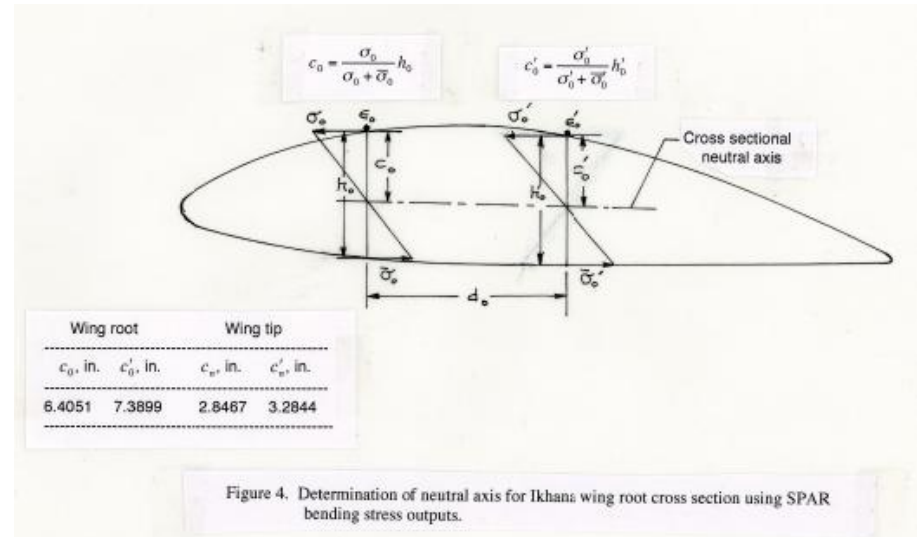
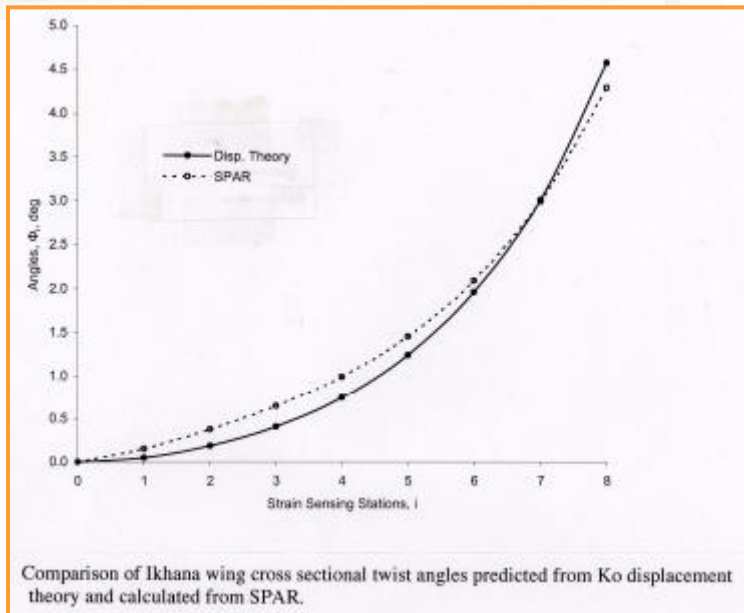


Figure 4. Determination of neutral axis for Ikhana wing root cross section using SPAR bending stress outputs.



Comparison of Ikhana wing cross sectional twist angles predicted from Ko displacement theory and calculated from SPAR.

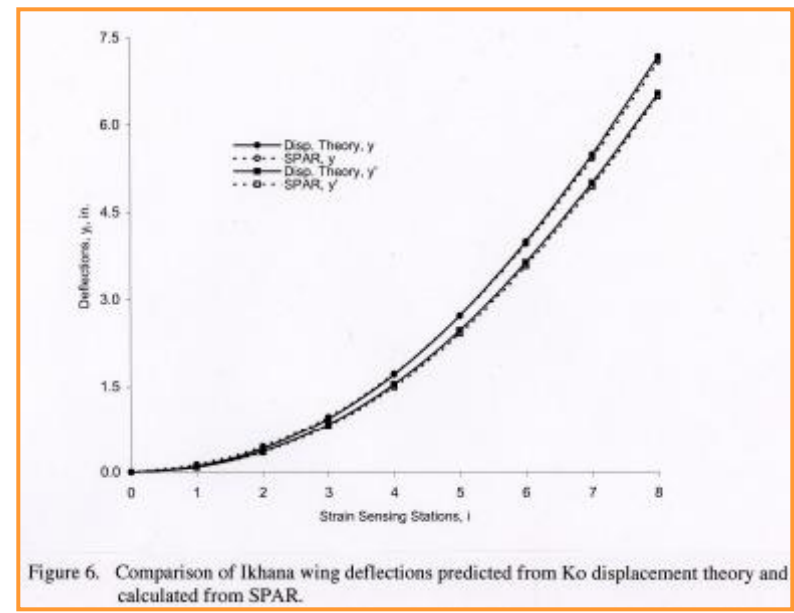


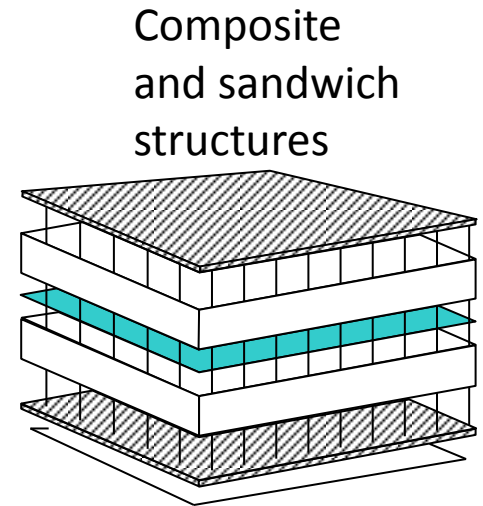
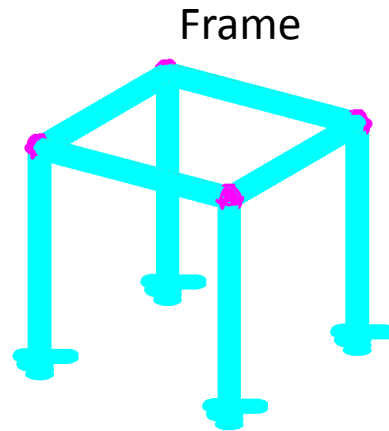
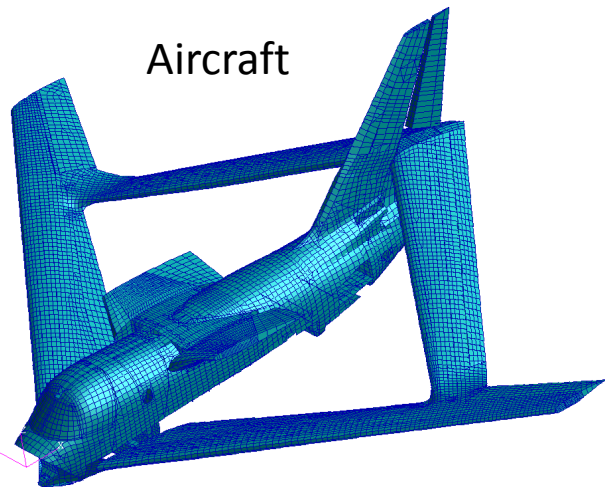
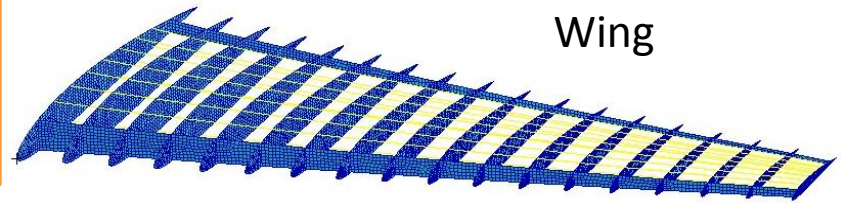
Figure 6. Comparison of Ikhana wing deflections predicted from Ko displacement theory and calculated from SPAR.

# NASA LaRC High-Fidelity, Full-Field Inverse FEM



From strains measured at discrete locations, determine full-field continuous displacements, strains, and stresses that represent the measured data with sufficient accuracy

$$\{ \mathbf{u}, \quad \boldsymbol{\varepsilon}, \quad \boldsymbol{\sigma}, \quad \mathbf{f}(\boldsymbol{\varepsilon}, \boldsymbol{\sigma}) = 0 \quad \mathbf{F}_{\text{ext}} \}$$



# Conceptual Framework of Inverse FEM: Discretized, high-fidelity solution



1. Discretization with iFEM:

– beam, plate, shell or solid  $\Omega^h$

2. Elements defined by a continuous displacement field  $\mathbf{u}^h(\mathbf{x})$

3. Strains defined by strain-displacement relations  $\boldsymbol{\varepsilon}^h = \mathbf{L} \mathbf{u}^h$  on  $\Omega^h$

4. Experimental strain-gauge data and iFE strains match up in a least-squares sense

$$\|\Delta \boldsymbol{\varepsilon}\|^2 = (\boldsymbol{\varepsilon}^h - \boldsymbol{\varepsilon}^\varepsilon)_{xi}^2$$

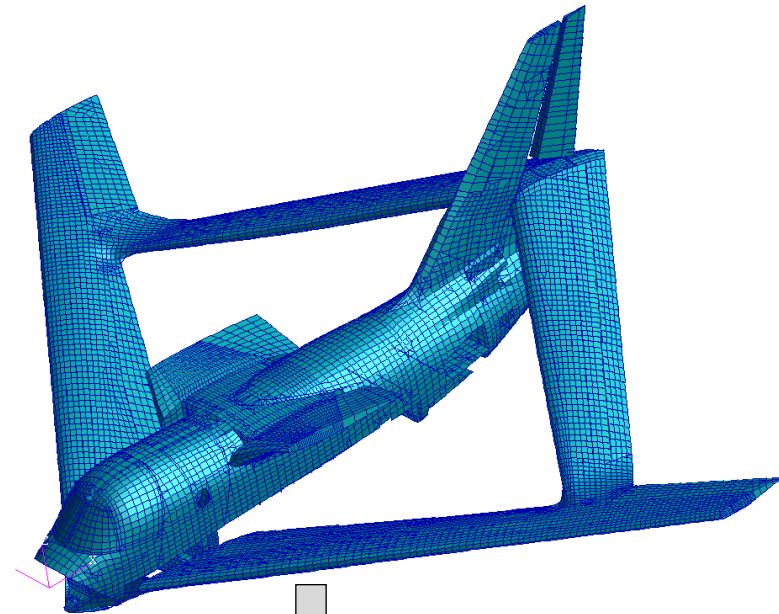
5. Displacement B.C.'s prescribed

$$\mathbf{u} = \bar{\mathbf{u}} \text{ on } \partial\Omega$$

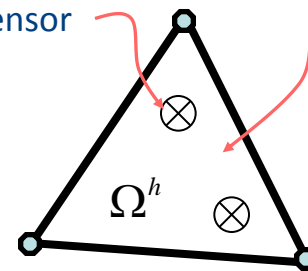
6. Linear algebraic Eqs determine nodal displacements

7. Element-level substitutions yield full-field strains, stresses (internal loads), and failure criteria

$$\boldsymbol{\sigma}^h = \mathbf{C} \boldsymbol{\varepsilon}^h \text{ on } \Omega^h$$



$\boldsymbol{\varepsilon}^\varepsilon$   
strain at sensor



$$\begin{aligned} \mathbf{u}^h &= \mathbf{u}^h(\mathbf{x}) \\ \boldsymbol{\varepsilon}^h &= \mathbf{L} \mathbf{u}^h \\ \boldsymbol{\sigma}^h &= \mathbf{C} \boldsymbol{\varepsilon}^h \end{aligned}$$

3-node inverse shell element

# First-Order Shear Deformation Theory: Flat inverse-shell element



- Kinematic assumptions account for deformations due to
  - Membrane
  - Bending
  - Transverse shear

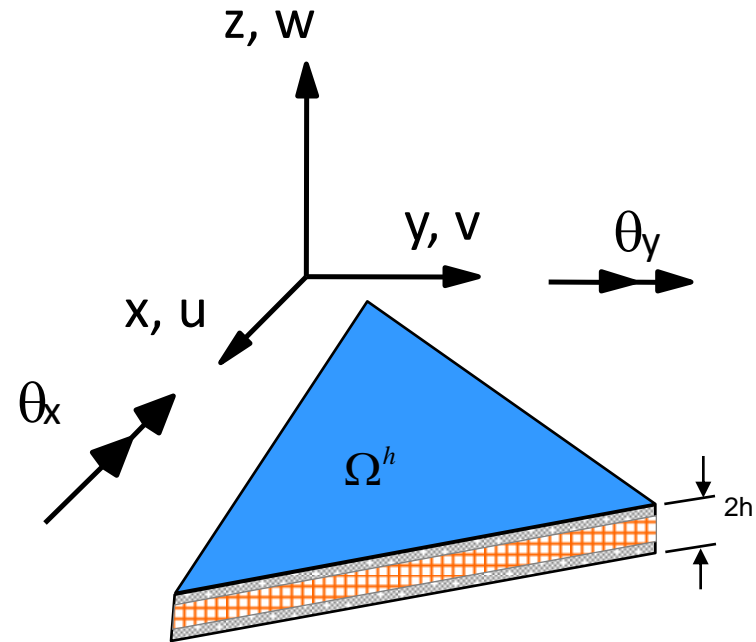
$$u_x(\mathbf{x}, t) = u + z \theta_y$$

$$u_y(\mathbf{x}, t) = v + z \theta_x$$

$$u_z(\mathbf{x}, t) = w$$

$$\mathbf{x} \equiv (x, y, z)$$

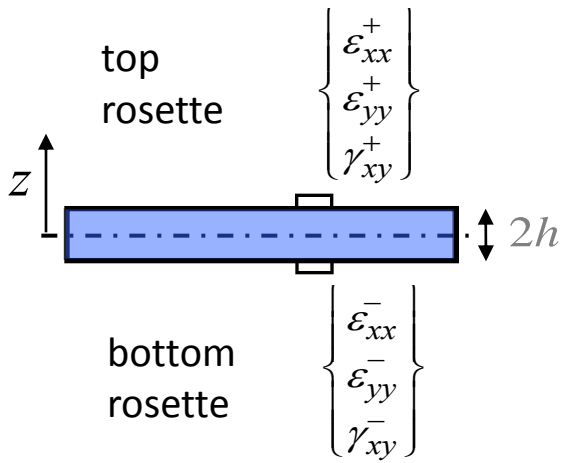
$$z \in [-h, h]$$



# Experimental in-situ strains

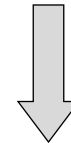


In-situ surface strains



Experimental strains via  
FSDT formalism

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}^\varepsilon \equiv \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix}^\varepsilon + z \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}^\varepsilon$$



Evaluate  
at  $x_i; z = \pm h$

$$\mathbf{e}_i^\varepsilon \equiv \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix}^\varepsilon = \frac{1}{2} \left( \begin{Bmatrix} \varepsilon_{xx}^+ \\ \varepsilon_{yy}^+ \\ \gamma_{xy}^+ \end{Bmatrix} + \begin{Bmatrix} \varepsilon_{xx}^- \\ \varepsilon_{yy}^- \\ \gamma_{xy}^- \end{Bmatrix} \right) \quad \mathbf{k}_i^\varepsilon \equiv \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}^\varepsilon = \frac{1}{2h} \left( \begin{Bmatrix} \varepsilon_{xx}^+ \\ \varepsilon_{yy}^+ \\ \gamma_{xy}^+ \end{Bmatrix} - \begin{Bmatrix} \varepsilon_{xx}^- \\ \varepsilon_{yy}^- \\ \gamma_{xy}^- \end{Bmatrix} \right)$$

# Full-Field Reconstruction using iFEM

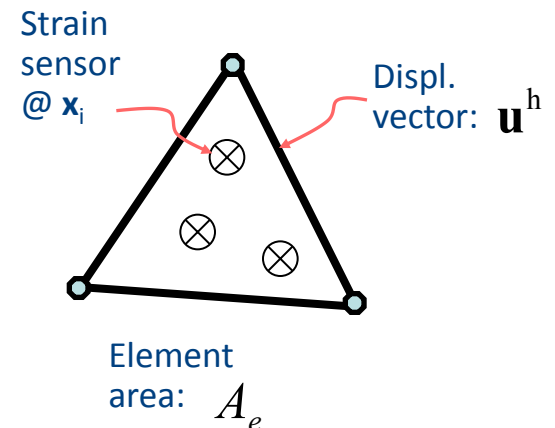


## Least-Squares variational formulation

- Plate formulation based on first-order shear deformation theory
  - Strain compatibility equations fulfilled
  - Strains treated as tensor quantities
  - No dependency on material, inertial or damping properties
- Efficient elements for
  - Beams and frames
  - Plates and shells
- Application to metal, multilayer composite, and sandwich structures

$$\Phi_e^p(\mathbf{u}^h) = p_1 \|\Delta \mathbf{e}\|^2 + p_2 \|\Delta \mathbf{k}\|^2 + p_3 \|\Delta \mathbf{g}\|^2$$

$p_i$  : Positive valued weighting constants  
– put different importance on the satisfaction of the individual strain components and their adherence to the measured data



# Discretization using iMIN3 elements

- Variational principle

$$\min : \sum_{e=1}^N \Phi_e^\lambda(\mathbf{u}^h) = 0$$

$\mathbf{A}(\mathbf{x}_i)$  symmetric, positive definite matrix (B.C.'s imposed)

$\mathbf{d}$  Nodal displacement vector

$\mathbf{b}$  r.h.s. vector, function of measured strain values

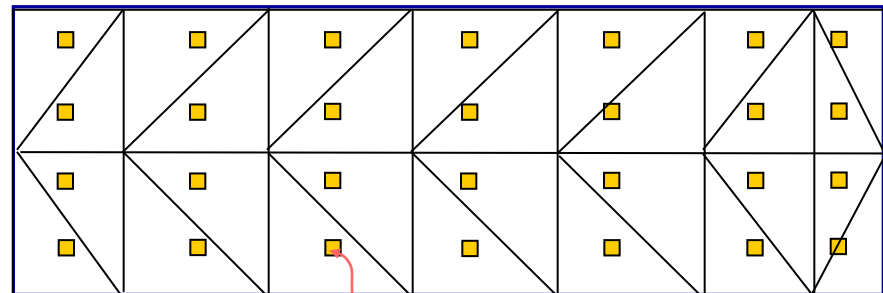
- Linear Eqs

$$\mathbf{A} \mathbf{d} = \mathbf{b}$$

- Efficient solution

$$\mathbf{d} = \mathbf{A}^{-1} \mathbf{b}$$

Coarse discretization sufficient  
(more efficient than direct FEM)



strain rosette



# Attributes of Inverse FEM



- **Theory**

- Strain-displacement relations fulfilled
- Least-squares compatibility with measured strain data
- Integrability conditions fulfilled
- Independent of material properties
- Stable solutions under small changes in input strain data (random error in measured strain data)
- Geometrically linear and nonlinear (co-rotational formulation) response
- Dynamic regime

- **Studies performed**

- Beam, frame, plate, and built-up shell structures
- Experimental studies using FBG strains and strain rosettes
- Transient dynamic response and strain data

- **Computational efficiency, architecture and modeling**

- Architecture as in standard FEM (e.g., user routine in ABAQUS)
- Superior accuracy on coarse meshes (advantage of integration)
- Beam, frame, plate, shell and built-up structures
- Thin and moderately thick regime
- Low and higher-order elements
- Use of partial strain data (over part of structure, or incomplete strain tensor data)

# iFEM applied to Plate Bending



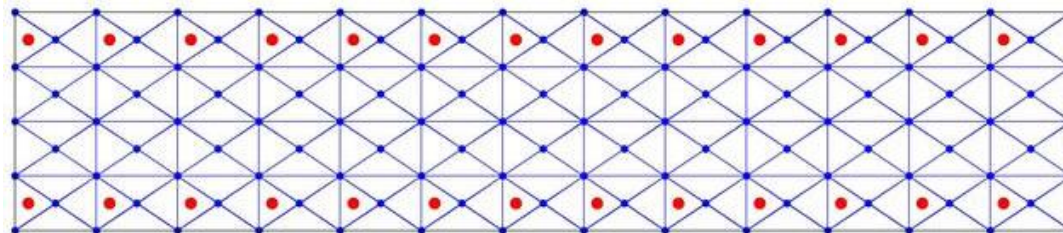
- Strain rosette data



- FBG sensors



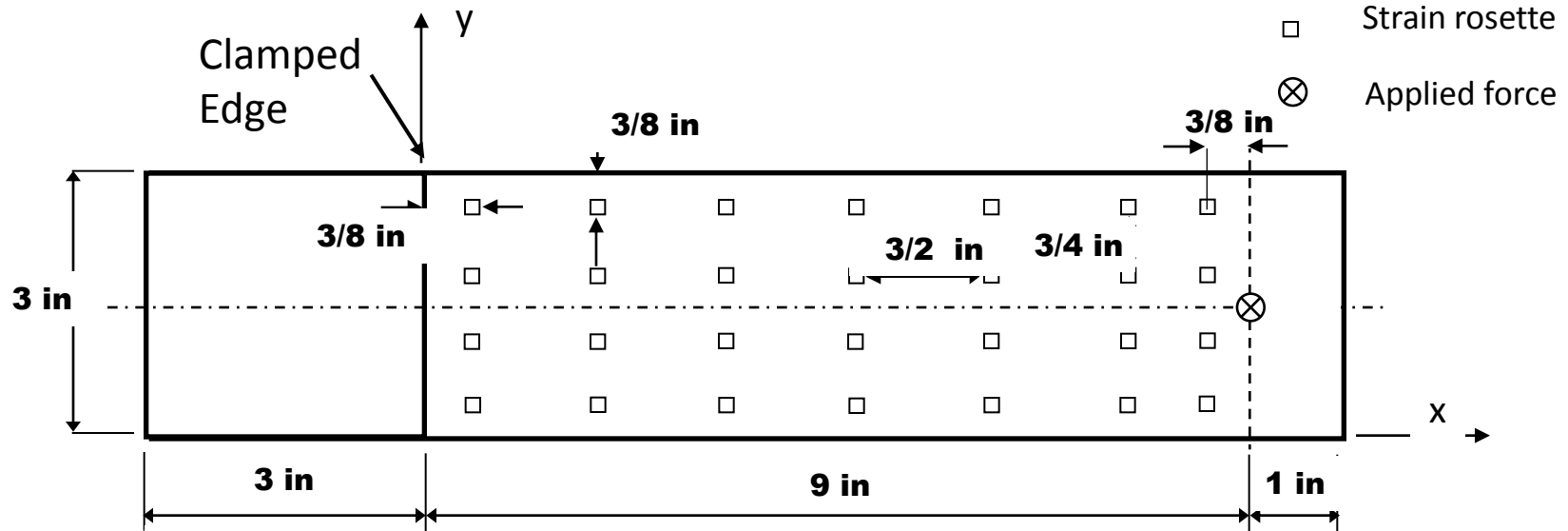
- Incomplete strain data



# Cantilevered Plate: iFEM using experimental strains



- Aluminum 2024-T3 alloy
  - Elastic modulus: 10.6 Msi
  - Poisson's ratio: 1/3
  - Thickness: 1/8 in
- Weight loaded at (9 in, 1.5 in)
  - $P = 5.784 \text{ lb}$  (2623 g)



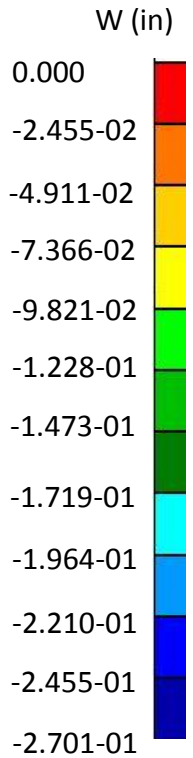
\* A. Tessler & J. Spangler. EWSHM (2004); P. Bogert et al., AIAA (2003)

# Deflection comparison

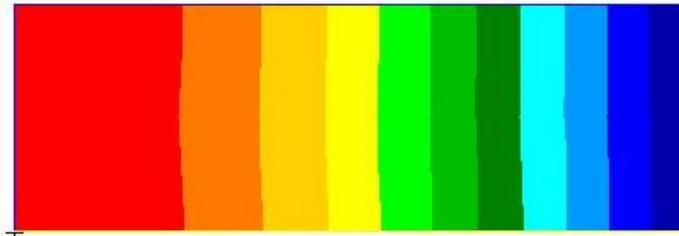
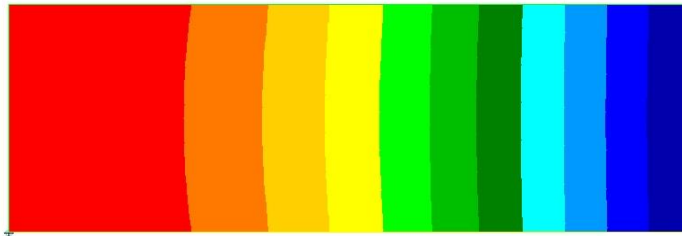
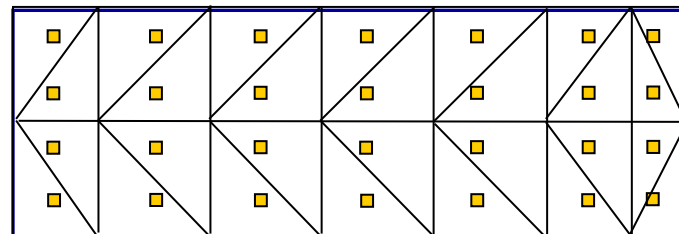
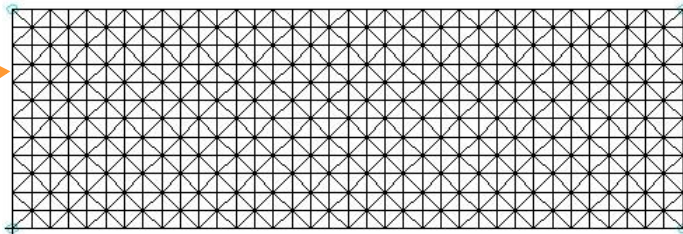
Measured deflection,  
 $W = 6.81 \text{ mm}$ ,  
at  $(214.3 \text{ mm}, 38.1 \text{ mm})$

FEM (ABAQUS)

Rossette strains



Clamped edge



Max. deflection  
 $W = 6.855 \text{ mm}$

Max. deflection  
 $W = 6.860 \text{ mm}$

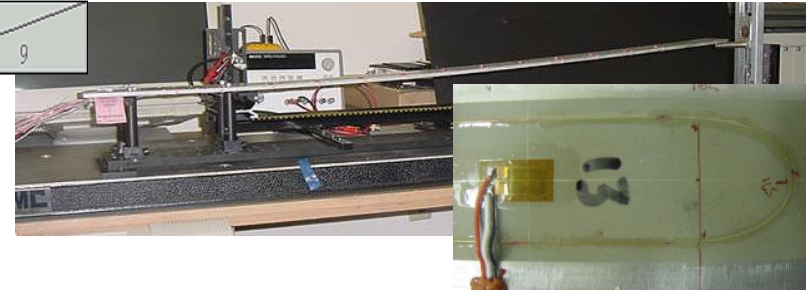
# Slender Beam Experiment using FOSS/iFEM\*



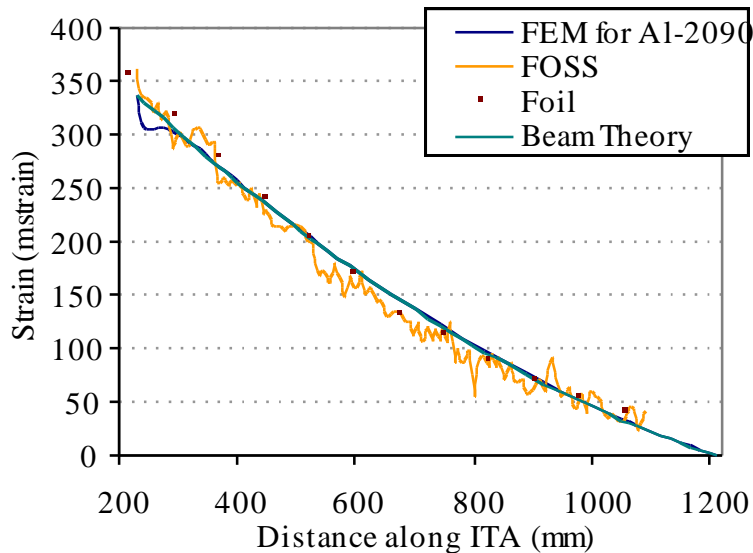
### Beam sensor layout and iFEM mesh



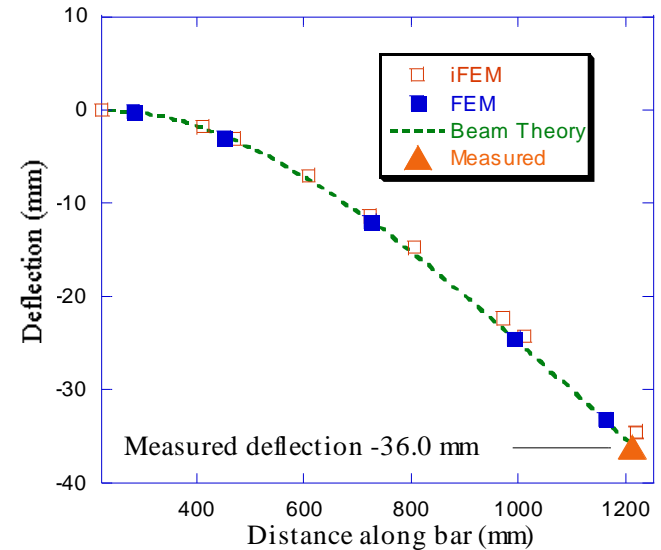
Cantilever beam instrumented with FOSS fiber.  
Deflection predicted from strain measurements via Inverse FEM.



### Measured and computed strain data

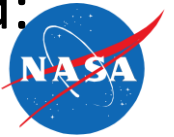


### Excellent correlation of deflection



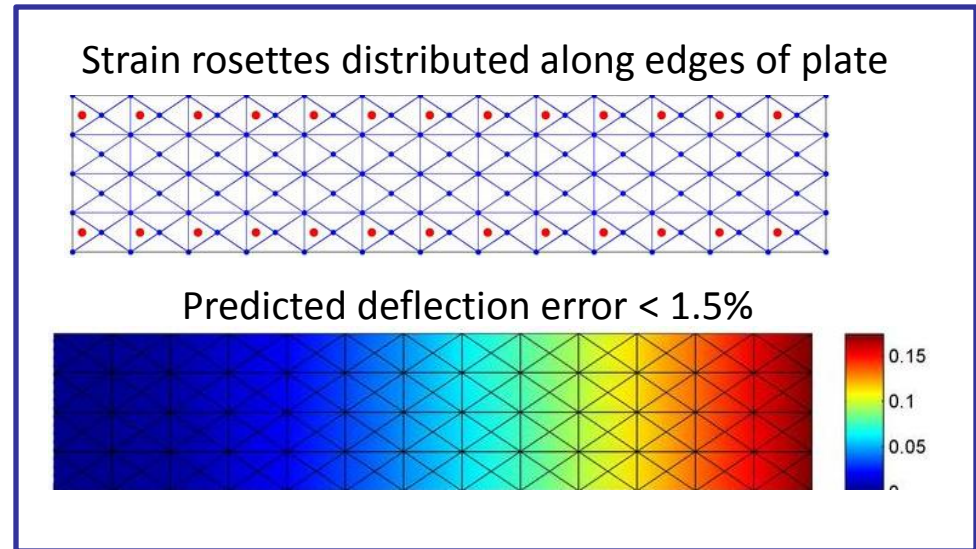
\* S. Vazquez et al., NASA-TM (2005)

# Cantilevered AL Plate in Bending under Uniform Load: Application of iFEM with Incomplete Strain Data



## Impact:

The iFEM is well-suited for real-time monitoring of the aircraft structural response and integrity when used in conjunction with advanced strain-measurement systems based on Fiber-Optic Bragg Grating strain sensors.



Predicts an accurate full-field deformation with a maximum deflection error of less than 1.5%. Only strain rosette measurements along panel edges are used in the analysis.

$$\Phi_e^p(\mathbf{u}^h) = p_1 \|\Delta \mathbf{e}\|^2 + p_2 \|\Delta \mathbf{k}\|^2 + p_3 \|\Delta \mathbf{g}\|^2$$

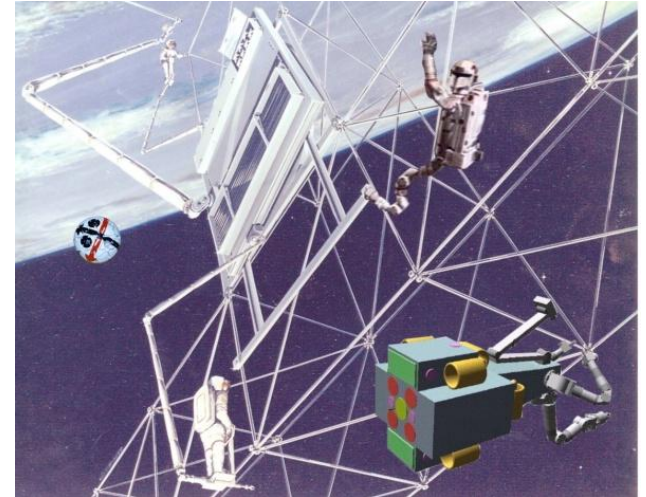
$p_i$  ( $i = 1, 2, 3$ ): weighting constants are set small  
in elements that do not have strain data

# 3-D Inverse Frame Finite Element Formulation\*

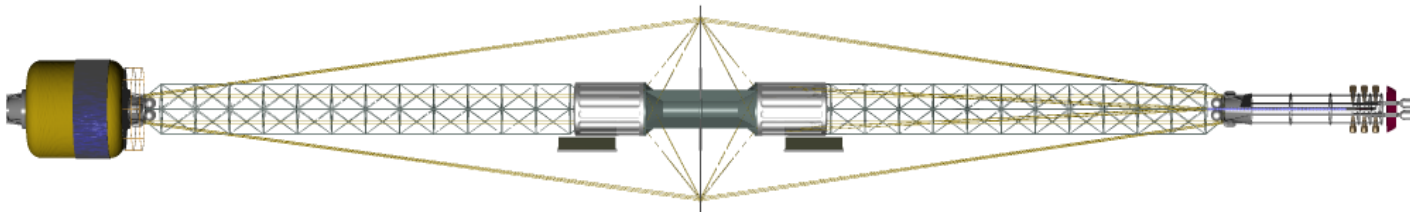


\* Collaboration with Politecnico di Torino

Power systems



Transfer vehicles

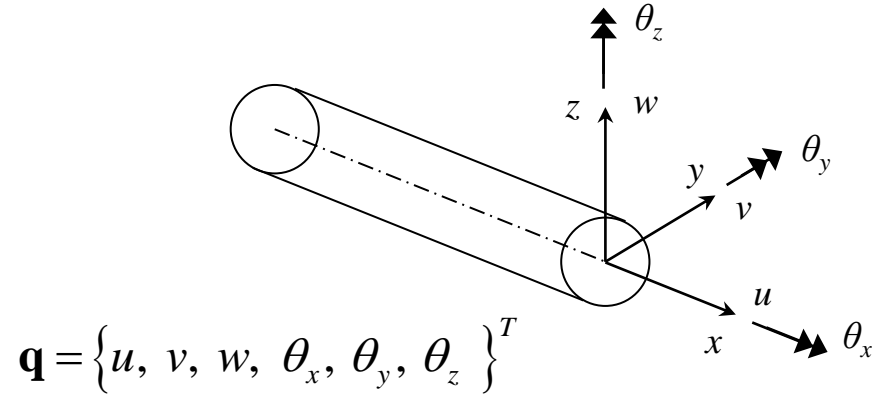


# 3-D Frame iFEM Formulation



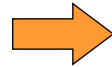
## Kinematic assumptions

$$\begin{cases} u_x(x, y, z) \equiv u(x) + z\theta_y(x) - y\theta_z(x) \\ u_y(x, y, z) \equiv v(x) - z\theta_x(x) \\ u_z(x, y, z) \equiv w(x) + y\theta_x(x) \end{cases}$$



- Strains

$$\begin{cases} \varepsilon_x = u_{,x} + z\theta_{y,x} - y\theta_{z,x} \\ \varepsilon_y = 0 \\ \varepsilon_z = 0 \\ \gamma_{xz} = (\theta_y + w_{,x}) + y\theta_{x,x} \\ \gamma_{yz} = 0 \\ \gamma_{xy} = (-\theta_z + v_{,x}) - z\theta_{x,x} \end{cases}$$



$$\begin{cases} \varepsilon_x = \varepsilon_{x0} + z\kappa_{y0} + y\kappa_{z0} \\ \gamma_{xz} = \gamma_{z0} + y\alpha_0 \\ \gamma_{xy} = \gamma_{y0} - z\alpha_0 \end{cases}$$

## Strain measures

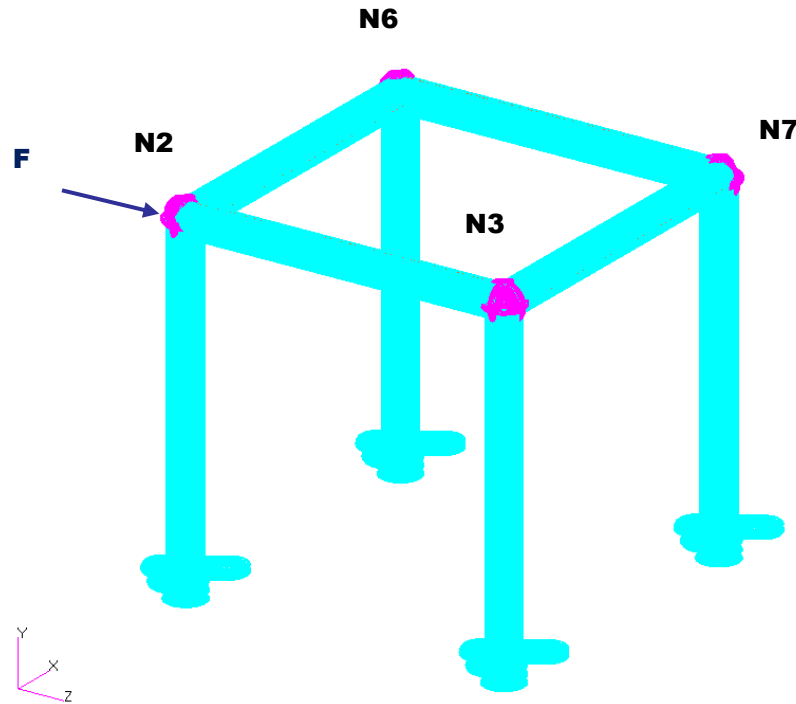
$$\mathbf{e}(\mathbf{q}) = \begin{Bmatrix} \varepsilon_{x0} \\ \kappa_{y0} \\ \kappa_{z0} \\ \gamma_{z0} \\ \gamma_{y0} \\ \alpha_0 \end{Bmatrix} \equiv \begin{Bmatrix} u_{,x} \\ \theta_{y,x} \\ -\theta_{z,x} \\ \theta_y + w_{,x} \\ -\theta_z + v_{,x} \\ \theta_{x,x} \end{Bmatrix}$$



# Numerical assessment



- Frame structure (thin-wall cross-section)

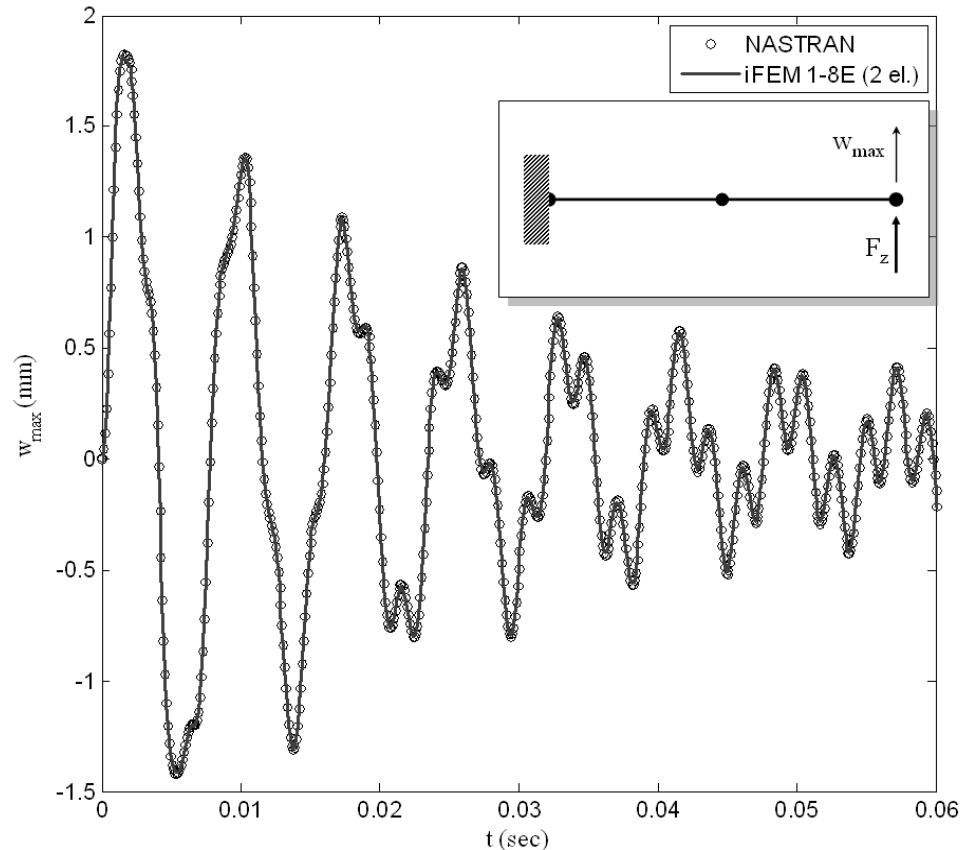


- Forward and Inverse FEM model data

Element type	NASTRAN (QUAD 4)	Inverse beam FE
No. of nodes	$3.29 \times 10^5$	8 (48 dofs)
No. of elem.	$3.28 \times 10^5$	8

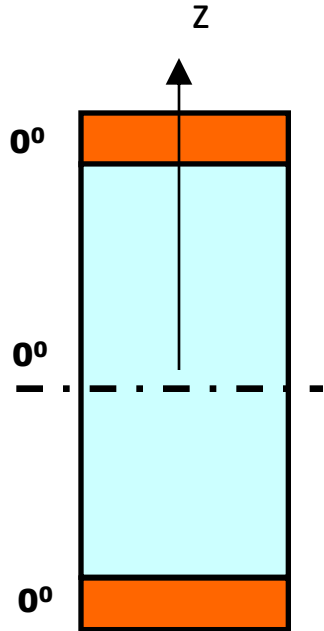
dof	iFEM/NASTRAN
$u_2$	1.008
$v_2$	1.002
$w_2$	1.002
$\theta_{x2}$	1.003
$\theta_{y2}$	1.007
$\theta_{z2}$	1.010
$u_3$	1.007
$v_3$	1.002
$w_3$	1.002
$\theta_{x3}$	1.003
$\theta_{y3}$	1.007
$\theta_{z3}$	1.010
$u_6$	1.008
$v_6$	1.001
$w_6$	0.996
$\theta_{x6}$	0.995
$\theta_{y6}$	1.007
$\theta_{z6}$	1.010
$u_7$	1.007
$v_7$	1.002
$w_7$	0.996
$\theta_{x7}$	0.995
$\theta_{y7}$	1.007
$\theta_{z7}$	1.009

# Transient response of damped cantilever beam: iFEM solution vs. high-fidelity NASTRAN model

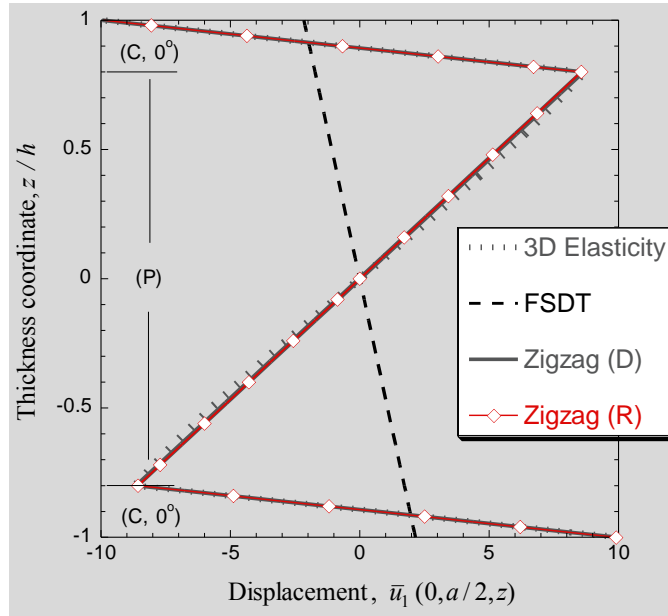


Tip beam deflection  $w_{\max}$  loaded by a transverse concentrated force  $F_z$  at  $f_0=1,400$  Hz

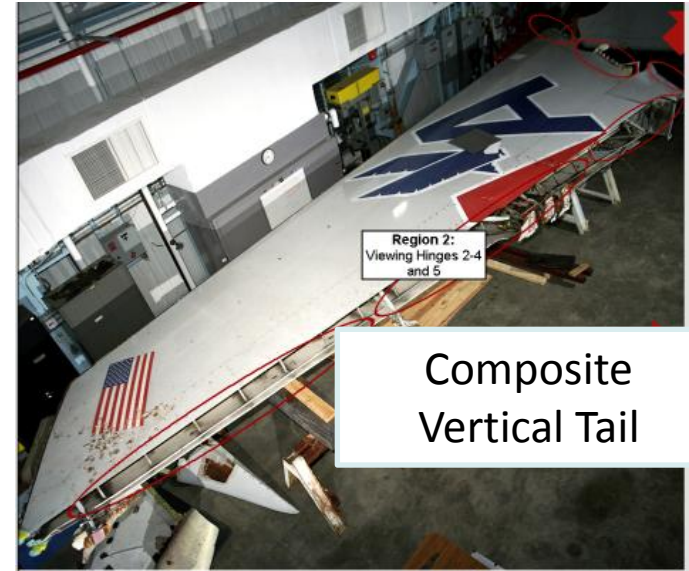
# iFEM based on Refined Zigzag Theory for Multilayered Composite and Sandwich Structures



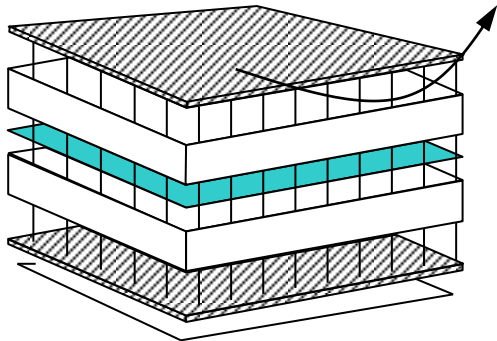
Zigzag kinematics



Airbus AA587



Composite Vertical Tail



Multifunctional Sandwich Panel

- Radiation shield
- Damage tolerant
- Thermal protection



# Summary

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- On-board structural integrity of nextgen aircraft, spacecraft, large space structures, and habitation structures
  - Safe, reliable, and affordable technologies
- Inverse FEM algorithms using FBG strain measurements
  - Real-time efficiency, robustness, superior accuracy
  - Large-scale, full-field applications
- Inverse FEM theory
  - Strain-displacement relations & integrability conditions fulfilled
  - Independent of material properties
  - Solutions stable under small changes in input data
  - Linear and nonlinear response



# Summary (cont'd)

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- Inverse FEM's architecture/modeling
  - Architecture as in standard FEM (user routine in ABAQUS)
  - Superior accuracy on coarse meshes
  - Frames, plates/shell and built-up structures
  - Thin and moderately thick regime
  - Low and higher-order elements
- Inverse FEM applications
  - Computational studies: frame, plate and built-up shell structures
  - Experimental studies: FBG strains and strain rosettes
  - Dynamic strain data
  - Zigzag theory for composites

# Collaborations & Interactions

- Lockheed Martin Co (J. Spangler)
- NASA LaRC (S. Vazquez, C. Quach, E. Cooper, and J. Moore)
- University of Hawaii (Prof. R. Riggs)
- Politecnico di Torino (Profs. Di Sciuva and Gherlone)

Ikhana fiber optic wing shape sensor team: clockwise from left, Anthony "Nino" Piazza, Allen Parker, William Ko and Lance Richards.



# Publications



- Tessler, A. and Spangler, J. L.: A Variational Principle for Reconstruction of Elastic Deformations in Shear Deformable Plates and Shells. NASA/TM-2003-212445 (2003).
- Tessler, A. and Spangler, J. L.: Inverse FEM for Full-Field Reconstruction of Elastic Deformations in Shear Deformable Plates and Shells. NASA/TM-2004-090744 (2004).
- Prosser\*, W. H., Allison, S. G., Woodard, S. E., Wincheski, R. A.; Cooper, E. G.; Price, D. C., Hedley, M., Prokopenko, M., Scott, D. A., Tessler, A., and Spangler, J. L.: Structural Health Management for Future Aerospace Vehicles. Proceedings of 2nd Australasian Workshop on Structural Health Monitoring (2004).
- Vazquez, S. L., Tessler, A., Quach, C. C., Cooper, E. G., Parks, J., and Spangler, J. L.: Structural Health Monitor Using High-density Fiber Optic Networks and Inverse Finite Element Method. Air Force Research Laboratory Integrated Systems Health Management Conference, August 17-19, 2004, Dayton, Ohio; Also NASA/TM-2005-213761 (2005).
- Tessler, A. and Spangler, J. L.: A Least-Squares Variational Method for Full-Field Reconstruction of Elastic Deformations in Shear-Deformable Plates and Shells. Computer. Methods Appl. Mech. Engrg. Vol. 194, 327-329 (2005).
- Gherlone, M., Mattone, M., Surace, C., Tassotti\*, A., and Tessler, A.: Novel Vibration-based Methods for Detecting Delamination Damage in Composite Plate and Shell Laminates. Key Engineering Materials, Vols. 293-294, 289-296 (2005).
- Tessler A.: Structural analysis methods for structural health management of future aerospace vehicles. NASA/TM-2007-214871 (2007).
- Gherlone M. Beam inverse finite element formulation. Politecnico di Torino, Oct. 2008.
- Cerracchio P., Gherlone M., Mattone M., Di Sciuva M., and Tessler A.: Inverse finite element method for three-dimensional frame structures. NASA/TP-2011, 2011.