National Aeronautics and Space Administration

Deformed Shape and Stress Reconstruction in Plate and Shell Structures Undergoing Large Displacements: Application of Inverse Finite Element Method using Fiber-Bragg-Grating Strains

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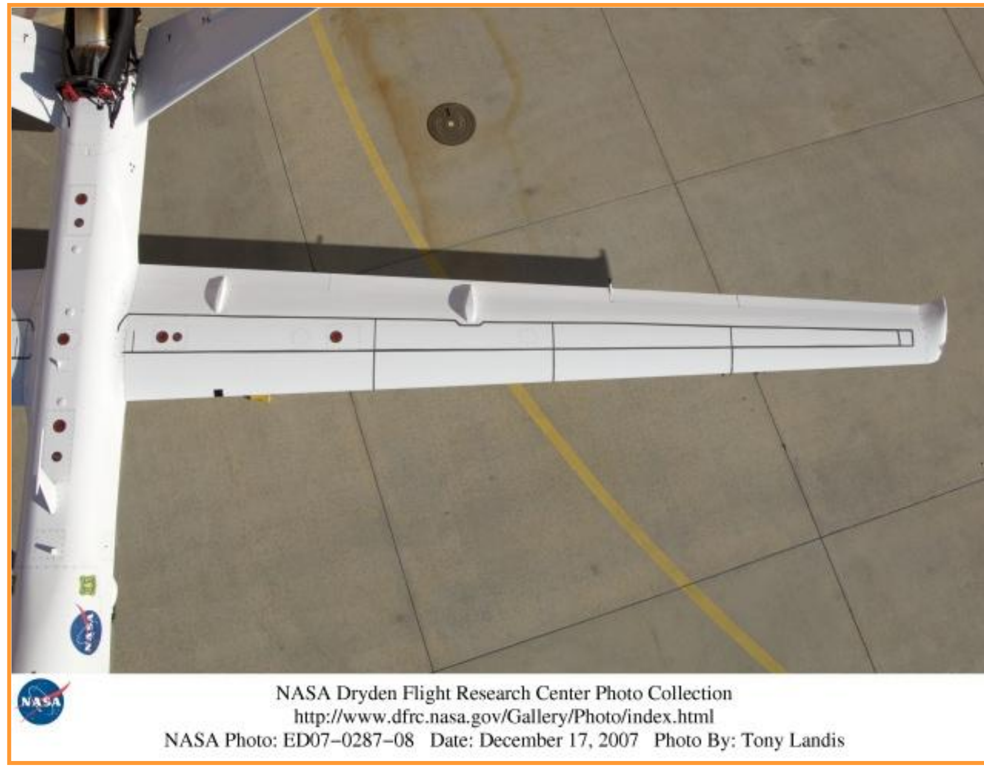
Inverse problems of wing deflection

Inverse FEM

- Using discrete strain measurements, **ε** $^{\varepsilon}$, determine <u>full-field solutions</u> for
  - displacements **u**
  - strains **ε** (**u**)
  - stresses **σ**(**u**)
- Ill-posed problem
- Uniqueness
- Stability

FBG (Fiber Bragg Grating) sensor is glued on top of wing to measure surface strain along axis (NASA Dryden)



Variational Formulation based on First-Order Shear Deformation Theory (Mindlin)

Kinematic Assumptions of First-Order Shear Deformation Theory

- Displacement components

$$u_x(\mathbf{x}, z) \equiv u(\mathbf{x}) + z\,\theta_y(\mathbf{x})$$

$$u_y(\mathbf{x}, z) \equiv v(\mathbf{x}) + z\,\theta_x(\mathbf{x})$$

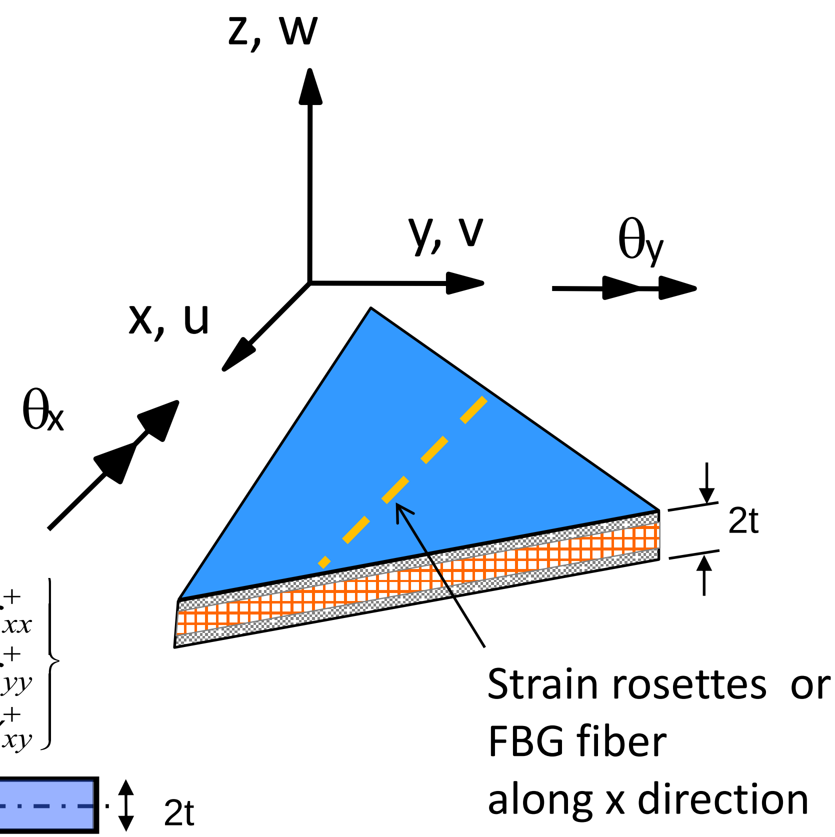
$$u_z(\mathbf{x}) \equiv w(\mathbf{x})$$

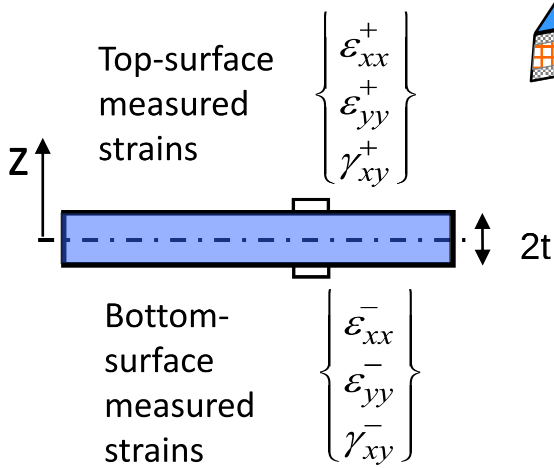
$\mathbf{x} \equiv (x, y)$ (strain-measurement directions)

$z \in [-t, t]$ (thickness coordinate)

- Deformations
  - Membrane
  - Bending
  - Transverse shear

Reference frame (aligned with strain-measurement directions)





Strains and Section Strains

- Inplane strains (=6)

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \equiv \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}$$

- Transverse-shear strains (=2)

$$\begin{Bmatrix} \varepsilon_7 \\ \varepsilon_8 \end{Bmatrix} \equiv \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial x} & 0 & 1 \\ 0 & 0 & \frac{\partial}{\partial y} & 1 & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \end{Bmatrix} \qquad \mathbf{g}(\mathbf{u}) = \mathbf{L}^s \, \mathbf{u}$$

- Section strains

3 membrane section strains

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} \equiv \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & 0 & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \end{Bmatrix} \qquad \mathbf{e}(\mathbf{u}) = \mathbf{L}^m \, \mathbf{u}$$

3 bending section strains

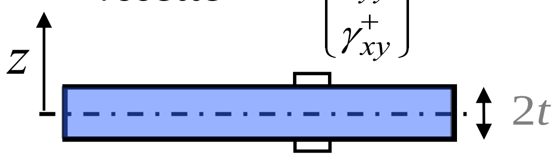
$$\begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \end{Bmatrix} \qquad \mathbf{k}(\mathbf{u}) = \mathbf{L}^b \, \mathbf{u}$$

Strain measurements relate to **membrane** & **bending** section strains

Surface strains measured at location $\mathbf{x}$

top rosette

$$\begin{Bmatrix} \varepsilon_{xx}^{+} \\ \varepsilon_{yy}^{+} \\ \gamma_{xy}^{+} \end{Bmatrix}$$



bottom rosette

$$\begin{Bmatrix} \varepsilon_{xx}^{-} \\ \varepsilon_{yy}^{-} \\ \gamma_{xy}^{-} \end{Bmatrix}$$

Express measured strains in terms of FSDT

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}^{\varepsilon} \equiv \begin{Bmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \end{Bmatrix}^{\varepsilon} + z \begin{Bmatrix} \varepsilon_{4} \\ \varepsilon_{5} \\ \varepsilon_{6} \end{Bmatrix}^{\varepsilon}$$

Evaluating at top and bottom ($z = \pm t$)

$$\mathbf{e}_i^{\varepsilon} \equiv \begin{Bmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \end{Bmatrix}^{\varepsilon} = \frac{1}{2}\left(\begin{Bmatrix} \varepsilon_{xx}^{+} \\ \varepsilon_{yy}^{+} \\ \gamma_{xy}^{+} \end{Bmatrix} + \begin{Bmatrix} \varepsilon_{xx}^{-} \\ \varepsilon_{yy}^{-} \\ \gamma_{xy}^{-} \end{Bmatrix} \right) \qquad \mathbf{k}_i^{\varepsilon} \equiv \begin{Bmatrix} \varepsilon_{4} \\ \varepsilon_{5} \\ \varepsilon_{6} \end{Bmatrix}^{\varepsilon} = \frac{1}{2t}\left(\begin{Bmatrix} \varepsilon_{xx}^{+} \\ \varepsilon_{yy}^{+} \\ \gamma_{xy}^{+} \end{Bmatrix} - \begin{Bmatrix} \varepsilon_{xx}^{-} \\ \varepsilon_{yy}^{-} \\ \gamma_{xy}^{-} \end{Bmatrix} \right)$$

$$\begin{Bmatrix} \varepsilon_{7} \\ \varepsilon_{8} \end{Bmatrix}^{\varepsilon}$$ Cannot be obtained from surface strains

iFEM variational formulation

Minimize an element functional $\Phi_e(\mathbf{u}^h)$ (a *weighted least-squares smoothing functional)* with respect to the unknown displacement degrees-of-freedom

$$\Phi_e(\mathbf{u}^h) = w_e \left\| \mathbf{e}\left(\mathbf{u}^h\right) - \mathbf{e}^\varepsilon \right\|^2 + w_k \left\| \mathbf{k}\left(\mathbf{u}^h\right) - \mathbf{k}^\varepsilon \right\|^2 + w_g \left\| \mathbf{g}\left(\mathbf{u}^h\right) - \mathbf{g}^\varepsilon \right\|^2$$

where the squared norms are

$$\left\| \mathbf{e}\left(\mathbf{u}^h\right) - \mathbf{e}^\varepsilon \right\|^2 \equiv \frac{1}{n} \int_{A_e} \sum_{i=1}^{n} \left[\mathbf{e}(\mathbf{u}^h)_i - \mathbf{e}_i^\varepsilon \right]^2 dx\, dy,$$

$$\left\| \mathbf{k}\left(\mathbf{u}^h\right) - \mathbf{k}^\varepsilon \right\|^2 \equiv \frac{(2t)^2}{n} \int_{A_e} \sum_{i=1}^{n} \left[\mathbf{k}(\mathbf{u}^h)_i - \mathbf{k}_i^\varepsilon \right]^2 dx\, dy,$$

$$\left\| \mathbf{g}\left(\mathbf{u}^h\right) - \mathbf{g}^\varepsilon \right\|^2 \equiv \frac{1}{n} \int_{A_e} \sum_{i=1}^{n} \left[\mathbf{g}(\mathbf{u}^h)_i - \mathbf{g}_i^\varepsilon \right]^2 dx\, dy$$

n Number of strain sensors per element

(w_e, w_k, w_g) Positive valued weighting constants associated with individual section strains (=8). They place different importance on the adherence of strain components to their measured values.

iFEM matrix equations

- Variational statement

$$\frac{\partial}{\partial \mathbf{u}_{\text{dof}}} \sum_{e=1}^{N} \Phi_e(\mathbf{u}^h) = 0$$

- Linear Eqs (displ. B.S.'s prescribed)

$$\mathbf{K}\,\mathbf{u}_{\text{dof}} = \mathbf{f}$$

- Displacement solution

$$\mathbf{u}_{\text{dof}} = \mathbf{K}^{-1}\mathbf{f}$$

$\mathbf{K}(\mathbf{x}_i)$	Symmetric, positive definite matrix
$\mathbf{u}_{\text{dof}}$	Nodal displacement vector
$\mathbf{f}(\boldsymbol{\varepsilon}^{\varepsilon})$	RHS vector, function of measured strain values

- iFEM integrates and smoothes strain data
- Higher accuracy than forward FEM

iFEM's selective, element-level (local) regularization

Important special cases

1. An element is missing measured transverse-shear section strains (standard case); Let $\alpha = 10^{-4}$ (small positive constant)

$$\left\| \mathbf{g}\left(\mathbf{u}^h\right) \right\|^2 \equiv \int_{A_e} \mathbf{g}(\mathbf{u}^h)^2 \, dx \, dy \quad (w_g = \alpha;\ w_e = w_k = 1)$$

2. An element is missing all measured section strains (in addition to (1))

$$\left\| \mathbf{e}\left(\mathbf{u}^h\right) \right\|^2 \equiv \int_{A_e} \mathbf{e}(\mathbf{u}^h)^2 \, dx \, dy \qquad (w_e = \alpha)$$

$$\left\| \mathbf{k}\left(\mathbf{u}^h\right) \right\|^2 \equiv (2t)^2 \int_{A_e} \mathbf{k}(\mathbf{u}^h)^2 \, dx \, dy \quad (w_k = \alpha)$$

3. An element is missing some measured-strain components

   – apply forms (2) to the missing components only

Simple and efficient inverse-shell element: iMIN3

- Anisoparametric interpolations
  (Tessler-Hughes, CMAME 1985)

  $(u, v, \theta_x, \theta_y)$: linear shape functions

  w: quadratic

- Section-strain fields

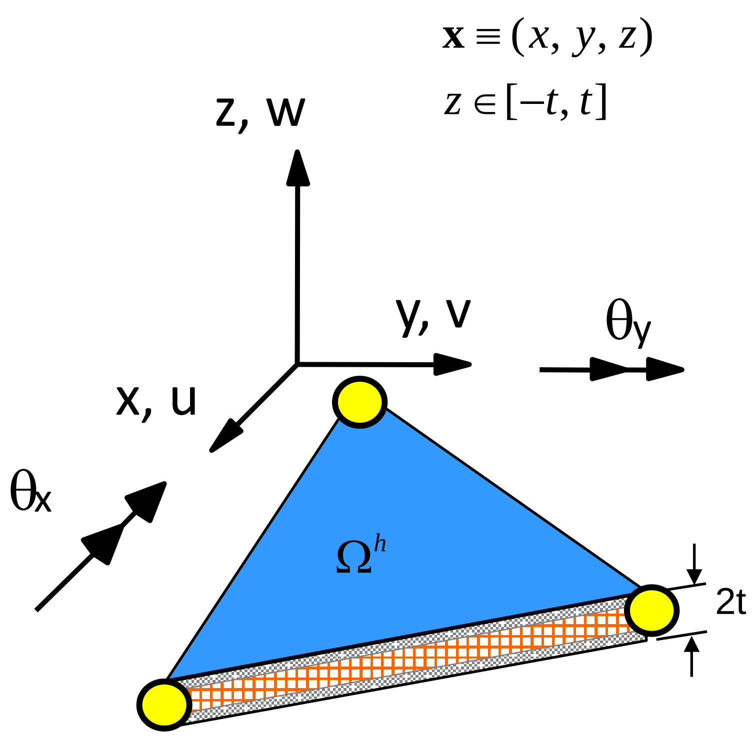
  $\mathbf{e}(\mathbf{u}^h), \mathbf{k}(\mathbf{u}^h)$: constant

  $\mathbf{g}(\mathbf{u}^h)$: linear

- 3 nodes, 5 or 6 dof/node

  $(u, v, w, \theta_x, \theta_y)$: 5 dof/plate

  $(u, v, w, \theta_x, \theta_y, \theta_z)$: 6 dof/shell



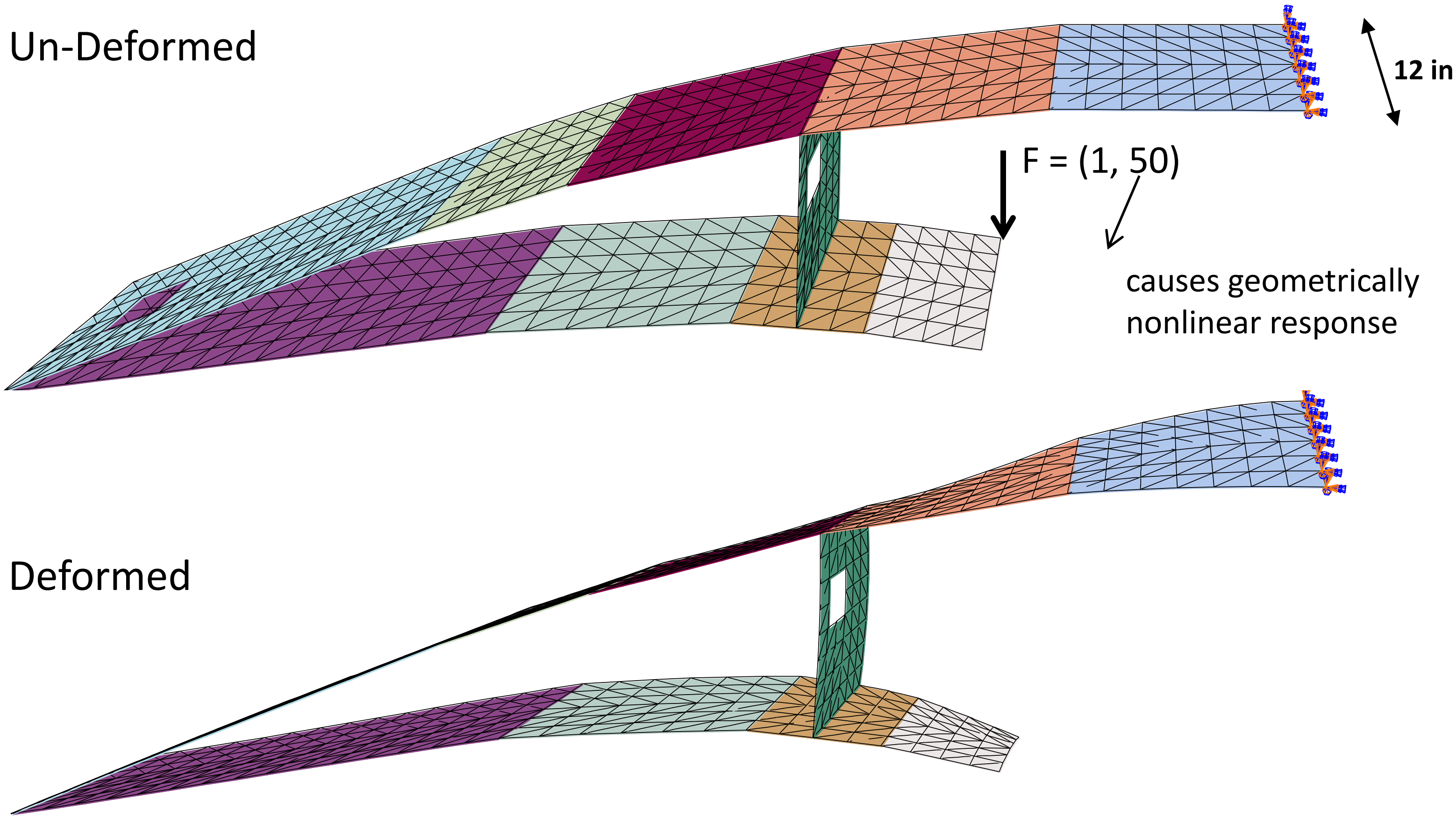
Demonstration problem

FEM shell model:
Aluminum stiffened flap with two rectangular cut-outs

Elastostatic deformations

(ABAQUS/STRI3 (Batoz) 3-node element, no shear deformation; 6 section strains only)

- Model has 10 planar element groups
- Each group has its own material reference frame to define strain orientations



3 iFEM modeling and stabilization schemes

Model A: Six FEM section strains are mapped onto all iFEM elements

- **One-to-one (high-fidelity)**
- **All elements have strain data but no shear strain measurements**

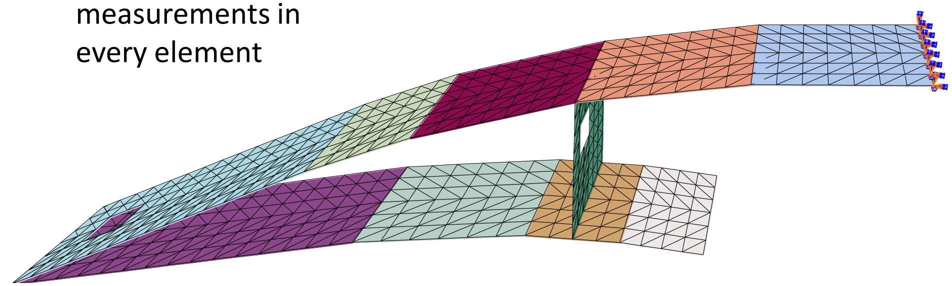
Model B: Six FEM section strains are mapped onto perimeter iFEM elements

- **Simulates tri-axial strain rosettes along the perimeter**
- **Interior elements have no strain data including the stiffener (local regularization)**

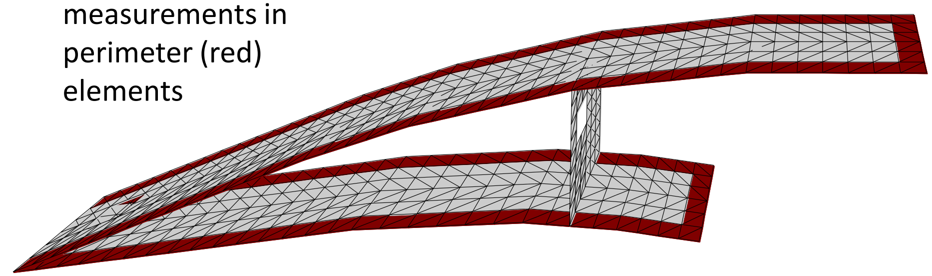
Model C: Two FEM section strains (axial) are mapped onto perimeter iFEM elements

- **Simulates linear strain gauges or FBG strain sensors**
- **Incomplete strain data**
- **Interior elements have no strain data including the stiffener (local regularization)**

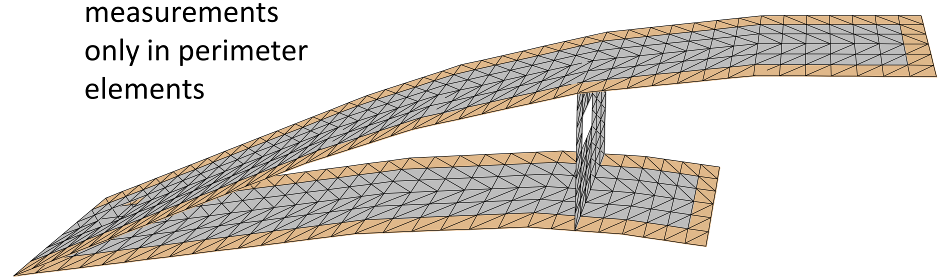
Tri-axial strain measurements in every element



Tri-axial strain measurements in perimeter (red) elements



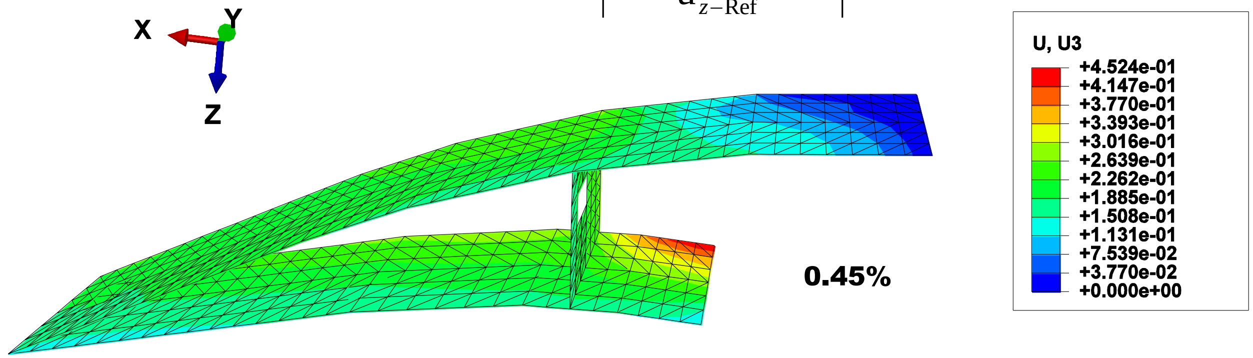
Axial strain measurements only in perimeter elements



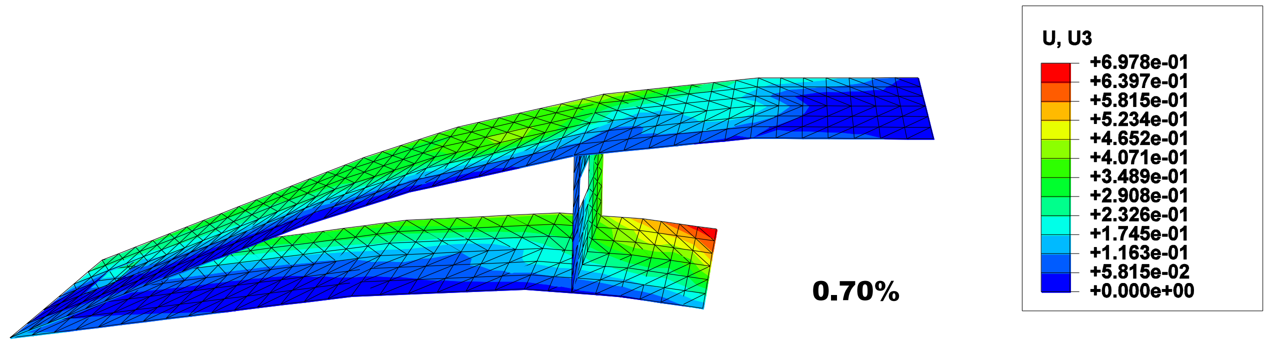
Linear problem: % error in reconstructed displacement, u_z

$$\%\ \text{Error}\ (u_z) = \left| \frac{u_{z-\text{Ref}\ i} - u_{z-\text{Est}\ i}}{u^{\text{Max}}_{z-\text{Ref}}} \right| \times 100$$

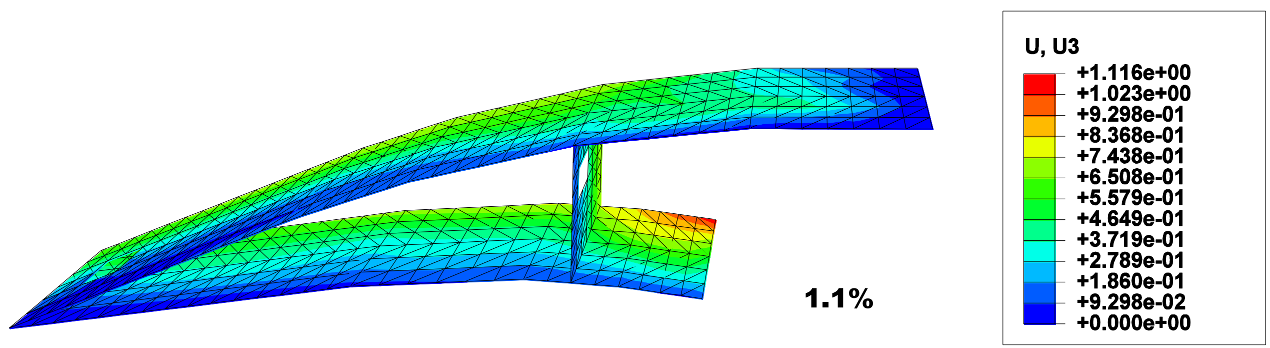
Model A: Six FEM section strains are mapped onto all iFEM elements



Model B: Six FEM section strains are mapped only onto perimeter iFEM elements



Model C: Two FEM section strains (axial) are mapped only onto perimeter iFEM elements



Linear problem: Deviations in u_z

Pearson correlation, r

iFEM model	0% noise in strains	5% noise in strains
A	1.00000	0.99998
B	0.99999	0.99998
C	0.99998	0.99985

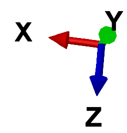
RMS

iFEM model	0% noise in strains	5% noise in strains
A	0.00017	0.00096
B	0.00020	0.00074
C	0.00035	0.00098

Mean % error

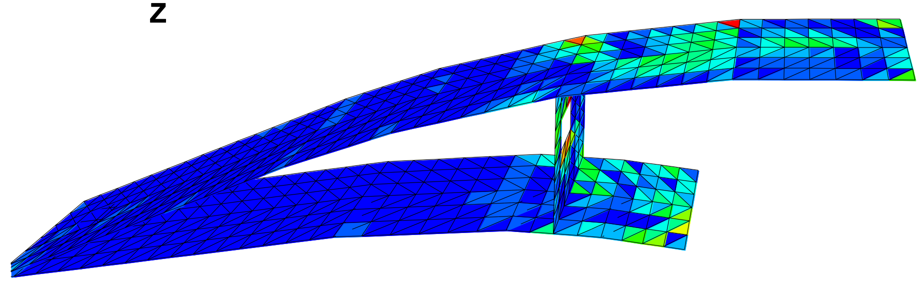
iFEM model	0% noise in strains	5% noise in strains
A	0.1951	1.0505
B	0.1934	0.8353
C	0.3575	1.0769

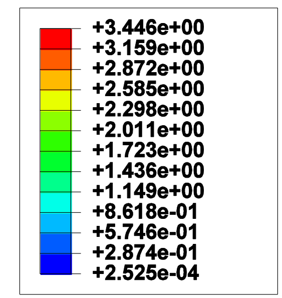
Linear problem: % error in reconstructed von Mises stress (bottom shell surface)



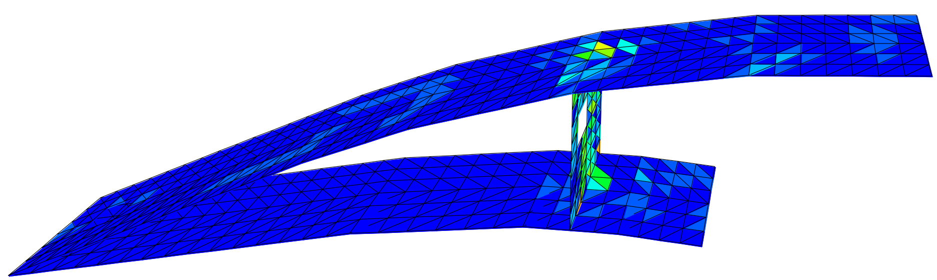
$$\% \text{ Error } (\sigma) = \left| \frac{\sigma_{\text{Ref i}} - \sigma_{\text{Est i}}}{\sigma_{\text{Ref}}^{\text{Max}}} \right| \times 100$$

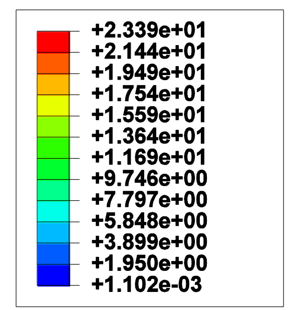
Model A: Six FEM section strains are mapped onto all iFEM elements



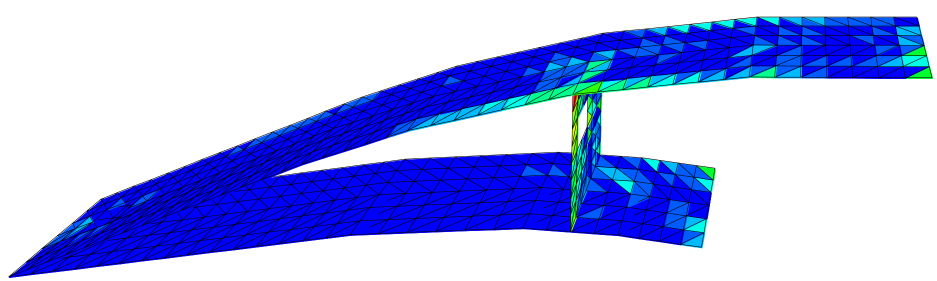


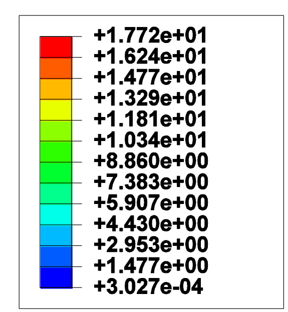
Model B: Six FEM section strains are mapped only onto perimeter iFEM elements





Model C: Two FEM section strains (axial) are mapped only onto perimeter iFEM elements





Linear problem: Deviations in von Mises stress

- Pearson correlation, r

$$r = \frac{\sum_{i=1}^{N}(\sigma_{\mathrm{Ref}\,i} - \bar{\sigma}_{\mathrm{Ref}})(\sigma_{\mathrm{EST}\,i} - \bar{\sigma}_{\mathrm{Est}})}{\sqrt{\sum_{i=1}^{N}(\sigma_{\mathrm{Ref}\,i} - \bar{\sigma}_{\mathrm{Ref}})^2}\sqrt{\sum_{i=1}^{N}(\sigma_{\mathrm{Est}\,i} - \bar{\sigma}_{\mathrm{Est}})^2}}$$

- Root-Mean-Square error

$$RMS = \sqrt{\frac{\sum_{i=1}^{N}(\sigma_{\mathrm{REF}\,i} - \sigma_{\mathrm{EST}\,i})^2}{N}}$$

- Mean % error

$$\frac{1}{N}\sum_{i=1}^{N}\left|\frac{\sigma_{\mathrm{Ref}\,i} - \sigma_{\mathrm{Est}\,i}}{\sigma_{\mathrm{Ref}}^{\mathrm{Max}}}\right| \times 100$$

Pearson correlation, r

iFEM model	0% noise in strains	5% noise in strains
A	0.9994	0.9993
B	0.9842	0.9844
C	0.9903	0.9896

RMS

iFEM model	0% noise in strains	5% noise in strains
A	4.0149	5.72724
B	21.9314	23.4736
C	15.7996	16.9735

Mean % error

iFEM model	0% noise in strains	5% noise in strains
A	0.3786	0.5553
B	1.6796	1.8404
C	1.3501	1.5133

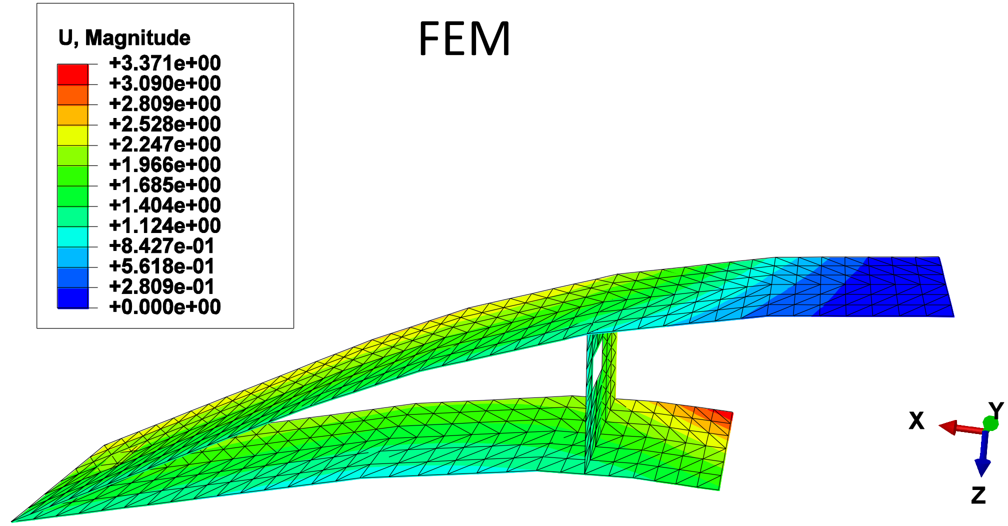
iFEM incremental algorithm for nonlinear deformations

- Use Nonlinear FEM as a virtual experiment (Lagrangean reference frame)
  - At each load increment of NL-FEM, compute the incremental section strains (6 components) that represent measured strain increments
  - Perform iFEM analysis using the strain increments to obtain the displacements and rotations
  - Update the geometry of iFEM mesh due to deformation using iFEM determined displacements, i.e., $\mathbf{x_1}=\mathbf{x_0}+\mathbf{u_1}$
  - Perform iFEM using the strain increments of the next load increment, and update the geometry for the next step $\mathbf{x_2}=\mathbf{x_1}+\mathbf{u_2}$

Nonlinear problem, F=50: Displacement magnitude (full load)

Reference: Nonlinear FEM/ABAQUS (STRI3)

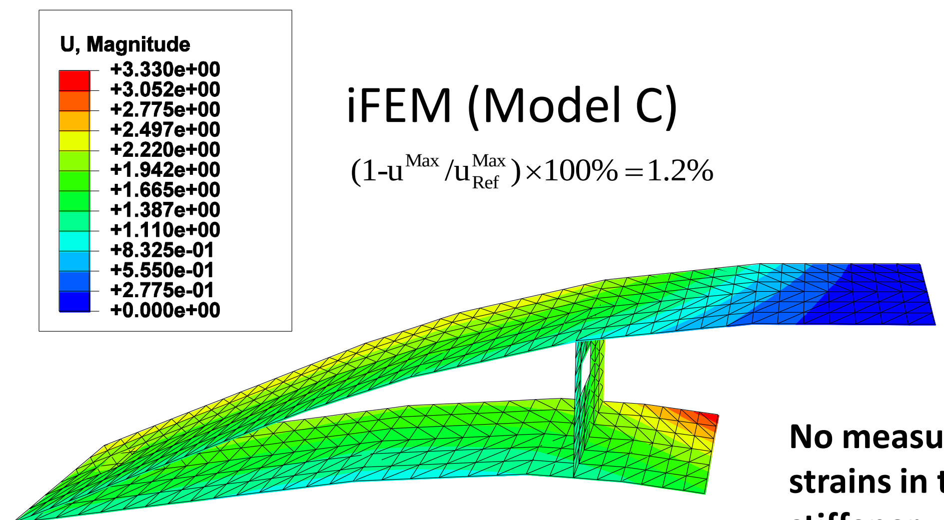
FEM



Model C: Two FEM section strains (axial) are mapped only onto perimeter iFEM elements (simulating FBG strains)

iFEM (Model C)

$$(1\text{-}u^{Max}/u^{Max}_{Ref})\times 100\% = 1.2\%$$



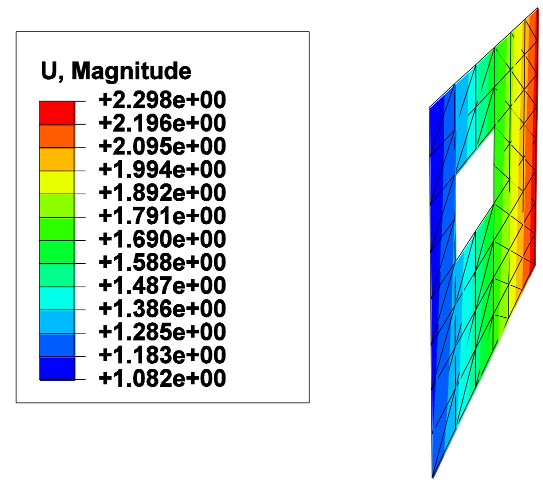
No measured strains in the stiffener

Nonlinear problem: Displacement magnitude (full load) of the stiffener

Reference: Nonlinear FEM/ABAQUS (STRI3)

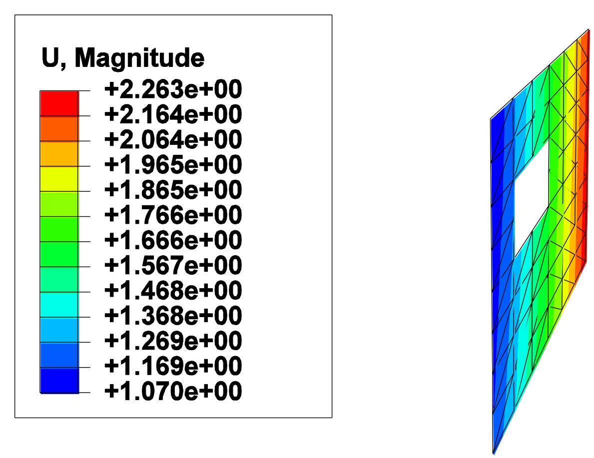
Model C: Two FEM section strains (axial) are mapped only onto perimeter iFEM elements (simulating FBG strains)

FEM



iFEM (Model C)

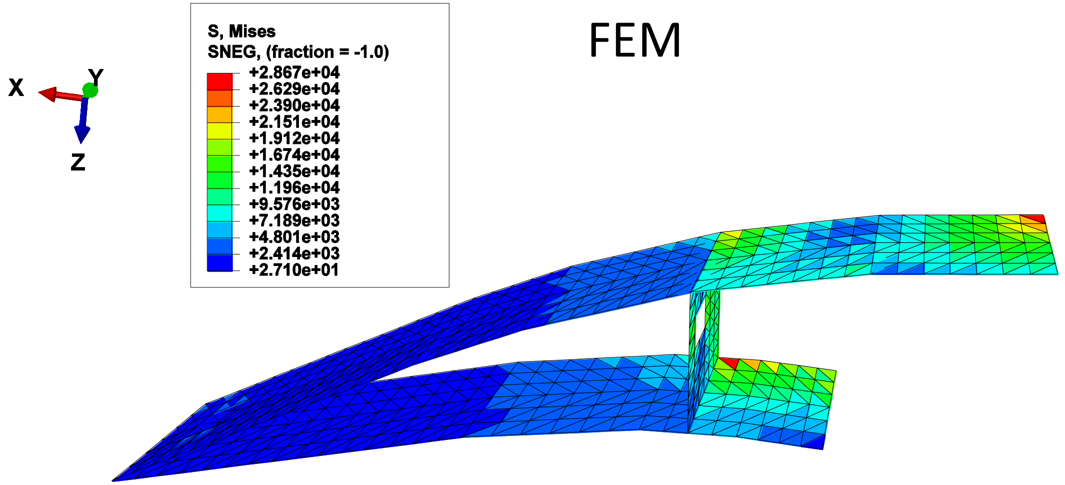
$$(1-u^{Max}/u^{Max}_{Ref})\times 100\% = 1.5\%$$



Nonlinear problem: Von Mises stress (full load)

Reference: Nonlinear FEM/ABAQUS (STRI3)

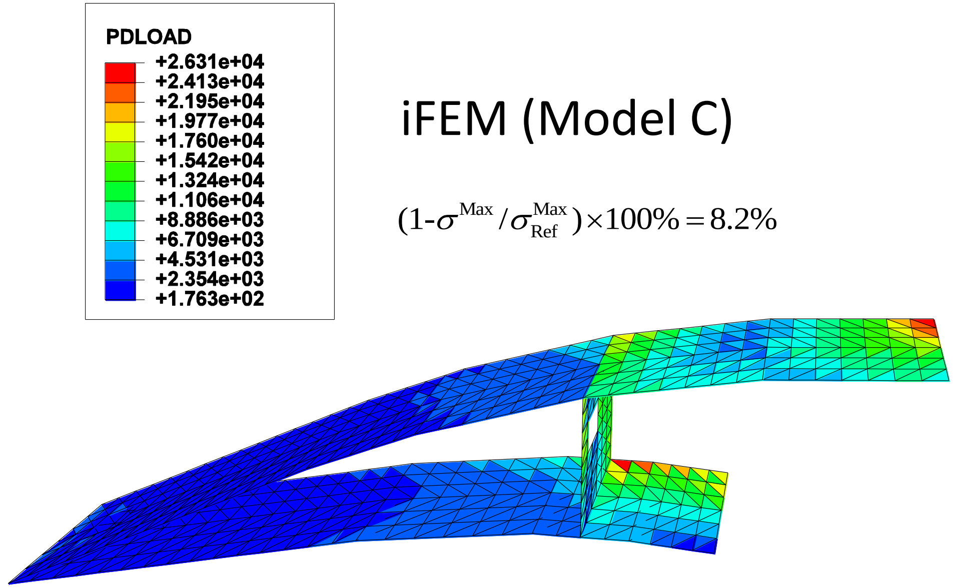
FEM



Model C: Two FEM section strains (axial) are mapped only onto perimeter iFEM elements (simulating FBG strains)

iFEM (Model C)

$$(1-\sigma^{\text{Max}}/\sigma^{\text{Max}}_{\text{Ref}})\times 100\% = 8.2\%$$



Summary

- On-board SHM of nextgen aircraft, spacecraft, large space structures, and habitation structures
  - Safe, reliable, and affordable technologies
- Inverse FEM algorithms for FBG strain measurements
  - Real-time efficiency, robustness, superior accuracy
  - Stable full-field solutions
- Inverse FEM theory
  - Strain-displacement relations & integrability conditions fulfilled
  - Independent of material properties
  - Solutions stable under small changes in input data
  - Linear and nonlinear response

Summary (cont'd)

- Inverse FEM's architecture/modeling
  - Architecture as in standard FEM (user routine in ABAQUS)
  - Superior accuracy on coarse meshes
  - Frames, plates/shell and built-up structures
  - Thin and moderately thick regime
  - Low and higher-order elements
- Inverse FEM applications
  - Computational studies: plate and built-up shell structures
  - Experimental studies with plates: FBG strains and strain rosettes

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